811

华南理工大学
2008年攻读硕士学位研究生入学考试试卷

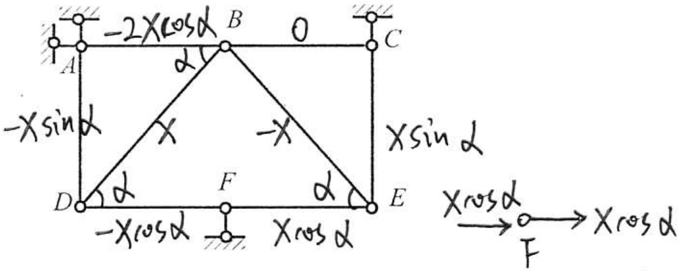
（请在答题纸上做答，试卷上做答无效，试后本卷必须与答题纸一同交回）

科目名称：结构力学

适用专业：工程力学，岩土工程，结构工程，防灾减灾工程及防护工程，桥梁与隧道工程

共 4 页

1、计算图示体系的自由度，试分析其体系的几何组成。（20分）



计算自由度：$W=6\times2-8-4=0$

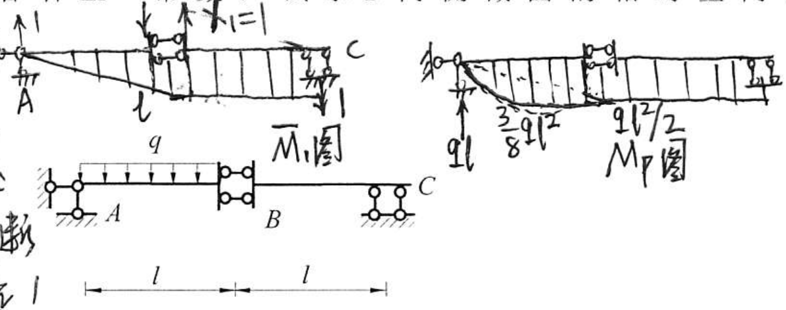
采用零载法，BD杆内力设为X

F结点平衡：$\Sigma F_{Fx}=0$ $X\cos\alpha+X\cos\alpha=0$ $X=0$

由此可知：图示体系各杆内力均为0，即无自内力存在，故体系自由度为0，为无多余约束几何不变体系。

2、图示结构各杆 $EI=$ 常数，试求 B 两侧截面的相对竖向位移。（20分）

首先根据无穷远处铰及无穷远处直线的性质，判断图示结构为静定！



分块图乘

$$\Delta_{yB}=\Sigma\int\frac{\overline{M}_1M_P}{EI}dx=\frac{1}{EI}\left(\frac{2}{3}\times l\times\frac{1}{8}ql^2\times\frac{l}{2}+\frac{1}{2}\times\frac{ql^2}{2}\times l\times\frac{2}{3}l\right)$$

$$+\left(l\times l\times\frac{1}{2}ql^2\right)\times\frac{1}{EI}=\frac{17ql^4}{24EI}$$

方法2.

系数和自由项：$\delta_{11}=\frac{1}{EI}(\frac{1}{2}\times1\times l\times\frac{2}{3})=\frac{l}{3EI}$

$\delta_{1k}=\frac{1}{lk}\times\frac{1}{l}=\frac{1}{l^2k}=\frac{l}{EI}$

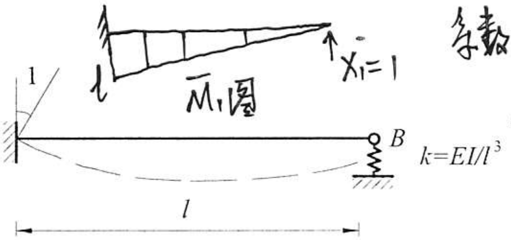
力法基本方程$(\delta_{11}+\delta_{1k})\cdot X_1=1$ 代入解得 $X_1=\frac{3EI}{4l}$

或者：$\delta_{11}=\frac{4l}{3EI}$

两步并作一步

由$M=X_1\cdot\overline{M}_1$可得 M图.

3、用力法求图示梁的M图。$EI=$常数，已知B支座的弹簧刚度为k。（20分）



系数和自由项：$\delta_{11}=\frac{1}{2EI}\times l\times l\times\frac{2}{3}l=\frac{l^3}{3EI}$

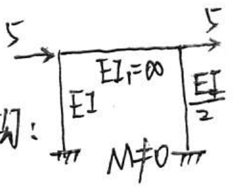
$\Delta_{1\theta}=-1\times l=-l$

力法基本方程：$\delta_{11}\cdot X_1+\Delta_{1\theta}=-\frac{X_1}{k}$ 代入后解得 $X_1=\frac{3EI}{4l^2}$

由 $M=X_1\cdot\overline{M}_1$ 可得 M图.

4、用位移法计算图示刚架，作M图。（15分）

方法2，取半结构：



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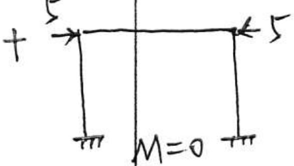


M=0

同样可解得横梁水平向侧移为$\frac{5l^2}{18i}$

系数和自由项：

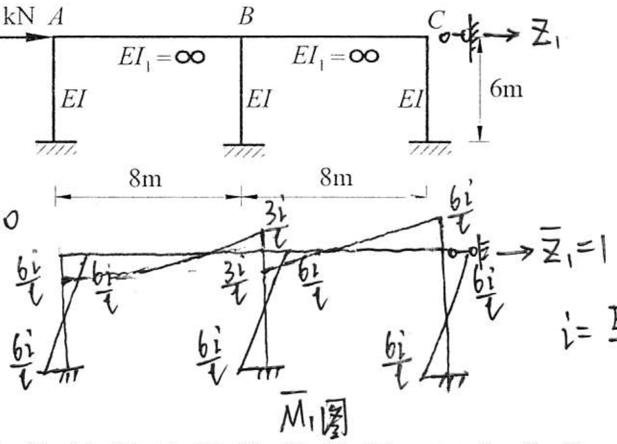
$r_{11}=3\times\frac{12i}{l^2}=\frac{36i}{l^2}$

$r_{1P}=-10kN$

基本方程：$r_{11}\cdot Z_1+r_{1P}=0$

代入解得$Z_1=\frac{+5l^2}{18i}$

$M=Z_1\cdot\overline{M}_1+\underbrace{M_P}_{=0}$



$i=\frac{EI}{l}$

5、用力矩分配法绘制图示梁的弯矩图。EI为常数。（计算二轮）（20分）

$i_1=\frac{EI_1}{l_1}=\frac{4E}{8}=\frac{E}{2}$ $i_2=\frac{2E}{8}=\frac{E}{4}$ $i_3=\frac{3E}{4}$

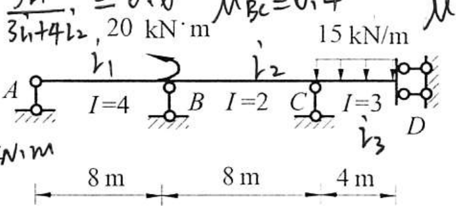
0.571 0.429

分配系数：$\mu_{BA}=\frac{3i_1}{3i_1+4i_2}=0.6$ $\mu_{BC}=0.4$ $\mu_{CB}=\frac{4i_2}{4i_2+i_3}=\frac{4}{7}$ $\mu_{CD}=\frac{3}{7}$

集中力偶计入AB杆，固端弯矩：

$M^F_{BA}=-20kN\cdot m$ $M^F_{CD}=-\frac{ql^2}{3}=-80kN\cdot m$

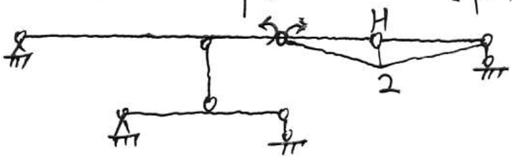
$M^F_{DC}=-\frac{ql^2}{6}=-40kN\cdot m$



杆端：	AB	BA	BC	CB	CD	DC
μ	/	0.6	0.4	$\frac{4}{7}$	$\frac{3}{7}$	/
M^F	0	-20	0	0	-80	-40
			22.84 ←	45.68	34.32 →	-34.32
		-1.704	-1.136 →	0.568		
			0.162 ←	0.324	0.244 →	-0.244
		-0.097	-0.065 →	0.033		
			0.01 ←	0.019	0.014 →	-0.014
M	0	-21.801	21.811	45.422	-45.422	-74.578

② M_E影响线：虚功方程：$\overline{M}_E\cdot\delta_{\theta E}+1\times\delta_P=0$ $\overline{M}_E=-\frac{\delta_P}{\delta_{\theta E}}$ 由此定出M_E影响线大致形状。现只须求 y_H 即可。

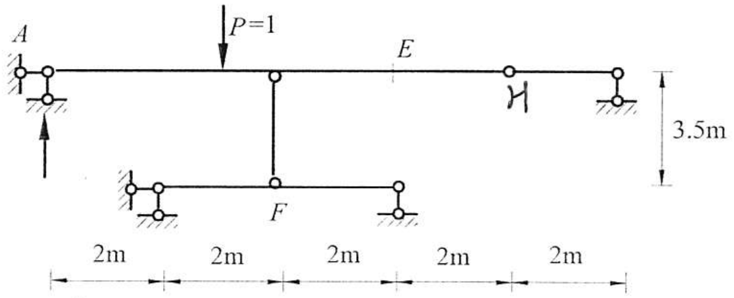
令P=1作用于点H，可求得 $y_H=2$



M_E影响线。

总结：作影响线用联合法，（有时直接简化为机动法）比较方便!!!

6、作图示结构的M_E 、M_F 影响线。（20分）



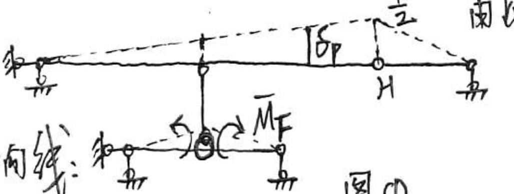
采用联合法作影响线：

① M_F影响线：下层梁结点F处变为铰接，虚功方程：$\overline{M}_F\cdot\delta_{\theta F}+1\times\delta_P=0$ $M_F=-\frac{\delta_P}{\delta_{\theta F}}$

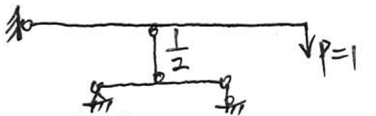
由此确定出 M_F影响线大致形状，现只须求 y_H 即可。

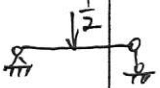
令P=1作用于H点，H点左边为基本结构，右边为附属。

M_F影响线：



图①



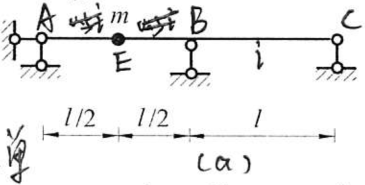


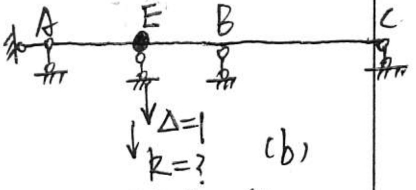
$y_H=\frac{\frac{1}{2}\times4}{4}=\frac{1}{2}$

7、求图示体系的自振频率。设$EI=$常数。（15分）

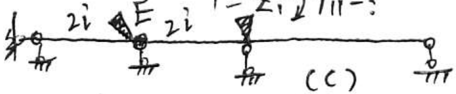
先判断结构为超静定，采用刚度法。

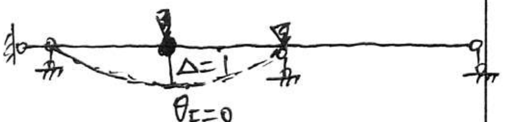
刚度求解：





图b质量处产生单位位移所需施加的力即为刚度k。采用位移法可解出k值。





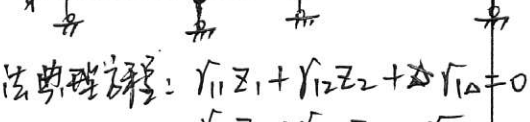
$r_{11}=8i+3i=11i$ $i=\frac{EI}{l}$

$r_{12}=r_{21}=4i$

$r_{22}=3\times2i+8i=14i$

$r_{1\Delta}=\frac{6\times2i}{\frac{l}{2}}=\frac{24i}{l}$

$r_{2\Delta}=\frac{24i}{l}-\frac{3\times2i}{\frac{l}{2}}=\frac{12i}{l}$



力法典型方程：$r_{11}Z_1+r_{12}Z_2+r_{1\Delta}=0$

$r_{21}Z_1+r_{22}Z_2+r_{2\Delta}=0$

$\begin{bmatrix}11i & 4i\\ 4i & 14i\end{bmatrix}$ $\begin{bmatrix}\frac{24i}{l}\\ -\frac{12i}{l}\end{bmatrix}$

代入解得 $Z_1=\frac{-48}{23l}$ $Z_2=-\frac{6}{23l}$

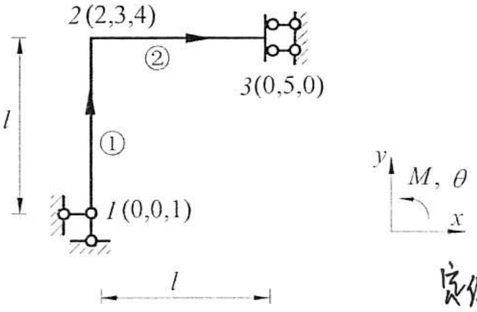
由此可求得 $M_{EA}=\frac{-312}{23l}i$ $M_{EB}=\frac{312}{23l}i$ $M_{BE}=\frac{144i}{23l}$

$k=\frac{-M_{EA}}{\frac{l}{2}}+\frac{M_{EB}+M_{BE}}{\frac{l}{2}}=\frac{1536i}{23l^2}$

振动方程：$-m\ddot{y}-ky=0$ $\ddot{y}+\frac{k}{m}y=0$

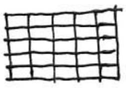
$$\omega=\sqrt{\frac{k}{m}}=8.17\sqrt{\frac{i}{ml^2}}=8.17\sqrt{\frac{EI}{ml^3}}$$

8、用先处理法写出图示刚架结构刚度矩阵的元素 $K_{22}, K_{34}, K_{15}, K_{45}$。$EI$，$EA$ 均为常数。（20分）



$$T=\left(\begin{array}{ccc:ccc}\cos\alpha & \sin\alpha & 0 & 0 & 0 & 0\\ -\sin\alpha & \cos\alpha & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ \hdashline 0 & 0 & 0 & \cos\alpha & \sin\alpha & 0\\ 0 & 0 & 0 & -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{array}\right)$$

定位向量 $\lambda^{①}=(0,0,1,2,3,4)^T$　$\lambda^{②}=(2,3,4,0,5,0)$

$\overline{K}^{①}=$  交叉处为数字 即单元刚度阵给出)

$$K^{①}=T^T\overline{K}^{①}T=\begin{array}{c|cccccc} & 0 & 0 & 1 & 2 & 3 & 4\\ \hline 0 & \frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} & -\frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2}\\ 0 & 0 & \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0\\ 1 & -\frac{6EI}{l^2} & 0 & \frac{4EI}{l} & \frac{6EI}{l^2} & 0 & \frac{2EI}{l}\\ 2 & -12EI/l^3 & 0 & 6EI/l^2 & 12EI/l^3 & 0 & 6EI/l^2\\ 3 & 0 & -\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0\\ 4 & -\frac{6EI}{l} & 0 & \frac{2EI}{l} & \frac{6EI}{l^2} & 0 & \frac{4EI}{l}\end{array}$$

$\alpha=90°$

$$其中\ T^{①}=\left(\begin{array}{ccc:ccc}0 & 1 & 0 & & & \\ -1 & 0 & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ \hdashline & & & 0 & 1 & 0\\ & 0 & & -1 & 0 & 0\\ & & & 0 & 0 & 1\end{array}\right)$$

附：

$$K^{②}=\overline{K}^{②}=\begin{array}{c|cccccc} & 2 & 3 & 4 & 0 & 5 & 0\\ \hline 2 & \frac{EA}{l} & 0 & 0 & \cancel{-\frac{EA}{l}} & 0 & \cancel{0}\\ 3 & 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} & \cancel{0} & -\frac{12EI}{l^3} & \cancel{\frac{6EI}{l^2}}\\ 4 & 0 & \frac{6EI}{l^2} & \frac{4EI}{l} & \cancel{0} & -\frac{6EI}{l^2} & \cancel{\frac{2EI}{l}}\\ 0 & \cancel{-\frac{EA}{l}} & \cancel{0} & \cancel{0} & \cancel{\frac{EA}{l}} & \cancel{0} & \cancel{0}\\ 5 & 0 & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & \cancel{0} & \frac{12EI}{l^3} & \cancel{-\frac{6EI}{l^2}}\\ 0 & \cancel{0} & \cancel{\frac{6EI}{l}} & \cancel{\frac{2EI}{l}} & \cancel{0} & \cancel{\frac{6EI}{l^2}} & \cancel{\frac{4EI}{l}}\end{array}$$

先处理法总体刚度矩阵：

$$K=\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5\\ \hline 1 & \frac{4EI}{l} & \frac{6EI}{l^2} & 0 & \frac{2EI}{l} & 0\\ 2 & \frac{6EI}{l^2} & \frac{12EI}{l^3}+\frac{EA}{l} & 0 & \frac{6EI}{l^2} & 0\\ 3 & 0 & 0 & \frac{EA}{l}+\frac{12EI}{l^3} & \frac{6EI}{l^2} & -\frac{12EI}{l^3}\\ 4 & \frac{2EI}{l} & \frac{6EI}{l^2} & \frac{6EI}{l^2} & \cancel{\frac{8EI}{l}}\ \frac{8EI}{l} & -\frac{6EI}{l^2}\\ 5 & 0 & 0 & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} & \frac{12EI}{l^3}\end{array}$$

$K_{22}=\frac{12EI}{l^3}+\frac{EA}{l}$　$K_{34}=\frac{6EI}{l^2}$

$K_{15}=0$　$K_{45}=\frac{-6EI}{l^2}$