2004 信号 研究生试题答案

一、

1. $\frac{\pi}{2}:\frac{\pi}{3}:\frac{\pi}{5}=15:10:6$

$T=m_1\frac{2\pi}{\omega_1}=15\cdot\frac{2\pi}{\frac{\pi}{2}}=60s$

$\Omega=\frac{2\pi}{T}=\pi/30\ rad/s$

2. $\frac{5}{2}:\frac{6}{5}:\frac{1}{7}=175:84:10$

$T=175\frac{2\pi}{\frac{5}{2}}=140\pi$ $\qquad\Omega=\frac{2\pi}{T}=\frac{2\pi}{140\pi}=\frac{1}{70}\ rad/s.$

二、

① $\int_{-\infty}^{t}e^{-\tau}\delta'(\tau)d\tau=\int_{-\infty}^{t}[e^{-0}\delta'(\tau)-(-1)e^{-0}\delta(\tau)]d\tau$

$=\int_{-\infty}^{t}[\delta'(\tau)+\delta(\tau)]d\tau=\delta(t)+u(t)$

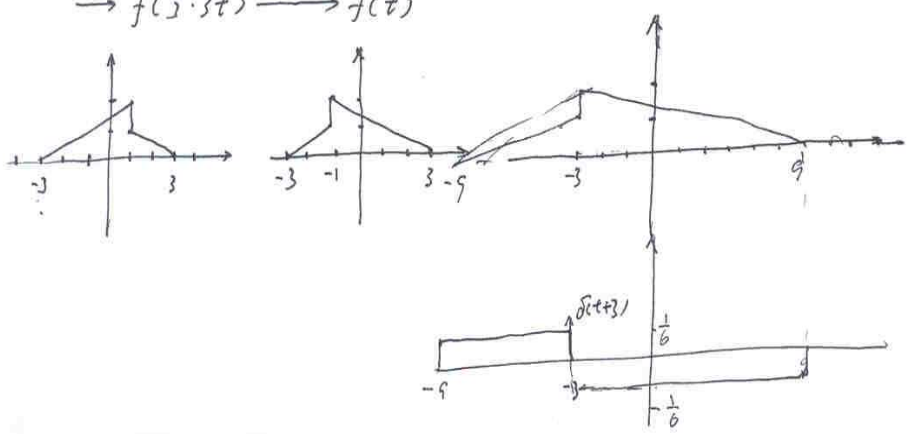
② $\int_{-\infty}^{\infty}\delta(at-4)f(t)dt=\frac{1}{|a|}\int_{-\infty}^{\infty}\delta(t-\frac{4}{a})f(t)dt=\frac{1}{|a|}f(\frac{4}{a})$

③ $u(k)*u(k-1)=u(k)*u(k)*\delta(k-1)=(k+1)u(k)*\delta(k-1)=ku(k-1)$

④ $\int_{-\infty}^{\infty}e^{-t}\delta''(t)dt=(-1)^2[e^{-t}]''_{|t=0}=1$

三、 $f(3-3t)\rightarrow f[-3(t-1+1)]\xrightarrow{左移1}f(-3t)\xrightarrow{反转}f(3t)\rightarrow$

$\rightarrow f(\frac{1}{3}\cdot3t)\xrightarrow{扩3倍}f(t)$



四. $f(t)=\frac{-1}{\pi t^2}$

① 解：$\because sgn(t) \to \frac{2}{j\omega}$ $\therefore \frac{2}{jt} \to 2\pi sgn(-\omega) \to \frac{1}{\pi t} \to -j\,sgn(\omega)$

$\because (\frac{1}{\pi t})' \to \frac{-1}{\pi t^2}$ $\therefore \frac{-1}{\pi t^2} \to j\omega[-j\,sgn(\omega)] = \omega\,sgn(\omega) = |\omega|$

②

$f(t)=tf(3t)$

解：$\because f(3t) \to \frac{1}{3}F(\frac{j\omega}{3})$

$-jtf(3t) \to \frac{d}{d\omega}[\frac{1}{3}F(\frac{j\omega}{3})]$ $\therefore \to tf(3t) \to \frac{j}{3}\frac{d}{d\omega}[F(\frac{j\omega}{3})]$

③

$t\frac{df(t)}{dt}$ —

解. $\because \frac{df(t)}{dt} \to j\omega F(j\omega)$

$\therefore t\frac{df(t)}{dt} \to j\frac{d}{d\omega}[j\omega \cdot F(j\omega)] = -[F(j\omega)+\omega\cdot\frac{d}{d\omega}F(j\omega)]$

④ $f(-t-2) \to F(j\omega)e^{-j2\omega} \to F(-j\omega)e^{j2\omega}$

⑤ $f(t)=[\frac{\sin(2\pi t)}{2\pi t}]^2$

解：$\because g_{4\pi}(t) \to 4\pi\frac{\sin 2\pi\omega}{2\pi\omega}$ $\therefore 4\pi\frac{\sin 2\pi t}{2\pi t} \to 2\pi g_{4\pi}(\omega)$

$\frac{\sin 2\pi t}{2\pi t} \to \frac{1}{2}\cdot g_{4\pi}(\omega)$ $\therefore [\frac{\sin(2\pi t)}{2\pi t}]^2 \to \frac{1}{2\pi}\frac{1}{2}g_{4\pi}(\omega) * \frac{1}{2}g_{4\pi}(\omega)$

$= \frac{1}{8\pi}g_{4\pi}(\omega) * g_{4\pi}(\omega) = \frac{1}{8\pi}\int_{-\infty}^{\infty}g_{4\pi}(\tau)g_{4\pi}(\omega-\tau)d\tau$

$= \frac{1}{8\pi}\int_{\omega-2\pi}^{2\pi}d\tau = (4\pi-\omega)\frac{1}{8\pi}$ $\omega>0$

$\to \frac{1}{8\pi}$ $\to \frac{1}{2}[1-\frac{|\omega|}{4\pi}]$

五、 $\frac{1}{2}\left[\frac{\sin^2(\frac{\omega+10}{2})}{(\frac{\omega+10}{2})^2}+\frac{\sin^2(\frac{\omega-10}{2})}{(\frac{\omega-10}{2})^2}\right]$

六、

① $u(t)-u(t-6)$

② $\because \delta''(t) \rightarrow (j\omega)^2=-\omega^2$

$\therefore \omega^2 \rightarrow -\delta''(t)$