武汉科技大学

2007 年硕士研究生入学考试试题答案

一、 计算题（每题 10 分，共 40 分）

1、$\int_0^{\infty} 4t\delta(t+1)dt = 0$

2、$\int_{-\infty}^{\infty} \frac{d}{dt}[\cos t\delta(t)]\sin tdt = \int_{-\infty}^{\infty} \delta'(t)\sin tdt = -\sin' t|_{t=0} = -\cos 0 = -1$

3、

$$[(1-t)\delta'(t)]*u(t) = [\delta'(t) - t\delta'(t)]*u(t) = [\delta'(t) - 0\delta'(t) + \delta(t)]*u(t)$$
$$= \delta'(t)*u(t) + \delta(t)*u(t) = \delta(t) + u(t)$$

4、

$$(-1)^k u(k)*(-1)^k u(k-1) = (-1)^k u(k)*(-1)(-1)^{k-1}u(k-1)$$
$$= -(-1)^k u(k)*(-1)^{k-1}u(k-1) = -(-1)^k u(k)*(-1)^k u(k)*\delta(k-1)$$
$$= -(k+1)(-1)^k u(k)*\delta(k-1) = -(k-1+1)(-1)^{k-1}u(k-1)$$
$$= -k(-1)^{k-1}u(k-1) = (-1)^k ku(k-1)$$

二、**求下列函数的傅立叶变换**（每题 10 分，共 30 分）

1、

$$\frac{\sin t}{t}$$

$$g_2(t) \to 2\frac{\sin\omega}{\omega}$$

$$2\frac{\sin t}{t} \to 2\pi g_2(-\omega) \qquad \frac{\sin t}{t} \to \pi g_2(\omega)$$

2、

$$f^2(t)\cos t$$
$$\cos t \to \pi[\delta(\omega+1) + \delta(\omega-1)]$$
$$f^2(t)\cos t \to \frac{1}{2\pi}F(\omega)*F(\omega)*\frac{1}{2\pi}\pi[\delta(\omega+1) + \delta(\omega-1)]$$
$$= \frac{1}{4\pi}F(\omega)*[F(\omega+1) + F(\omega-1)]$$

3、

$tu(t)$

$u(t) \to \pi\delta(\omega) + \dfrac{1}{j\omega}$

$-jtu(t) \to \dfrac{d}{d\omega}\left[\pi\delta(\omega) + \dfrac{1}{j\omega}\right] = \pi\delta'(\omega) - \dfrac{1}{j\omega^2}$

$tu(t) \to j\pi\delta'(\omega) - \dfrac{1}{\omega^2}$

三、**求下列函数的傅立叶逆变换**

1、$\delta'(\omega)$

$\Theta\ \delta'(t) \to j\omega$

$\therefore jt \to 2\pi\delta'(-\omega) = -2\pi\delta'(\omega) \qquad \delta'(\omega) \to -j\dfrac{t}{2\pi}$

2、$\pi g_4(\omega)$

$\Theta\ g_4(t) \to 4Sa(2\omega) \qquad \therefore 4Sa(2t) \to 2\pi g_4(-\omega) = 2\pi g_4(\omega)$

$\pi g_4(\omega) \to \dfrac{2Sa(2t)}{\pi}$

3、

$F(j\omega) = [u(\omega) - u(\omega-4)]e^{-j\omega} = [g_4(\omega-4)]e^{-j\omega} = [g_4(\omega-4)]e^{-j(\omega-4)}e^{-j4}$

$g_4(t) \to 4Sa(2\omega)$

$4Sa(2t) \to 2\pi g_4(-\omega) = 2\pi g_4(\omega)$

$[g_4(\omega-4)] \to \dfrac{2Sa(2t)}{\pi}e^{j4t}$

$[g_4(\omega-4)]e^{-j(\omega-4)}e^{-j4}$

令ω-4=ϖ, 则$[g_4(\varpi)]e^{-j\varpi}e^{-j4} \to \dfrac{2Sa[2(t-1)]}{\pi}e^{-j4}$

考虑ω-4=ϖ 则$[g_4(\omega-4)]e^{-j(\omega-4)}e^{-j4} \to \dfrac{2Sa[2(t-1)]}{\pi}e^{-j4} \bullet \mathrm{e}^{\mathrm{j}4\mathrm{t}} = \dfrac{2Sa[2(t-1)]}{\pi}e^{-j4(t-1)}$

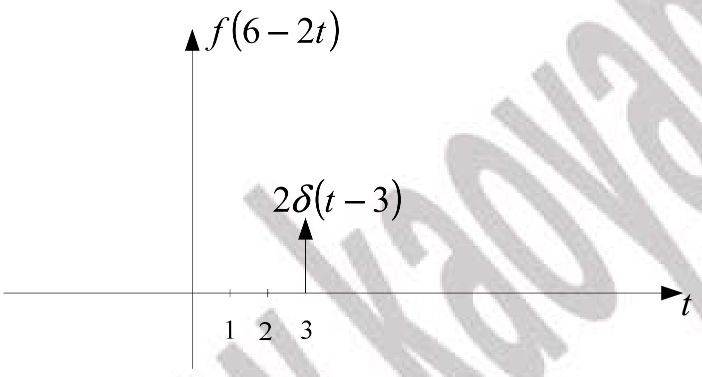
也可以:$\Theta\ [g_4(\omega-4)] \to \dfrac{2Sa(2t)}{\pi}e^{j4t} \qquad \therefore [g_4(\omega-4)]e^{-j\omega} \to \dfrac{2Sa[2(t-1)]}{\pi}e^{-j4(t-1)}$

四、 一个脉冲：$u(t)-u\left(t-\frac{T}{2}\right)\to\frac{1}{s}-\frac{1}{s}e^{-\frac{T}{2}s}=\frac{1}{s}\left(1-e^{-\frac{T}{2}s}\right)$

一串脉冲：等比数列 $\frac{1-e^{-\frac{T}{2}s}}{s\left(1-e^{-Ts}\right)}$

第二串脉冲： $\frac{1-e^{-\frac{T}{2}s}}{s\left(1-e^{-Ts}\right)}-\frac{1-e^{-\frac{T}{2}s}}{s\left(1-e^{-Ts}\right)}e^{-\frac{T}{2}s}$

五、 已知信号 $f(6-2t)=2\delta(t-3)$，求 $\int_{-1}^{\infty}f(t)dt$



$$f(6-2t)=f[-2(t-3)]\to f[-2(t-3+3)](\text{左移})\to f(-2t)(\text{对折})$$

$$\to f(2t)(\text{扩展一倍})\to f\left(\frac{1}{2}\times 2t\right)=f(t)=4\delta(t)$$

$$\int_{-1}^{\infty}4\delta(t)dt=4$$