理论力学.

2008解答

一、解

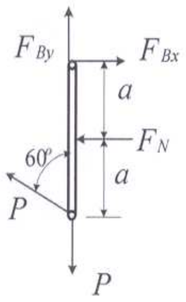
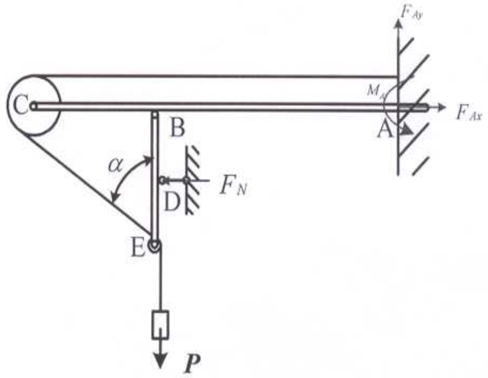


图 8分



1　以BE杆为研究对象

$\sum M_B = 0$

$P\cos 30^o \times 2a - F_N \times a = 0$　5分

$\therefore F_N = \sqrt{3}P$

2　以整体为研究对象

$\sum F_x = 0$

$F_{Ax} + P - F_N = 0$　4分

$\therefore F_{Ax} = F_N - P = (\sqrt{3}-1)P = 0.732P$

$\sum F_y = 0$

$F_{Ay} - P = 0$　4分

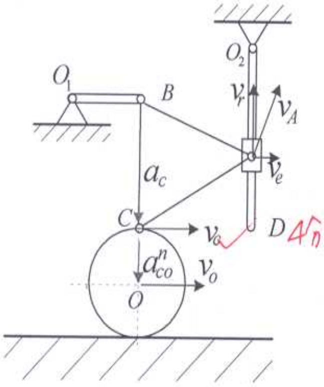
$\therefore F_{Ay} = P$

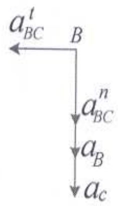
$\sum M_A = 0$

$M_A - P\times r - F_N \times a + P\times 2l = 0$　4分

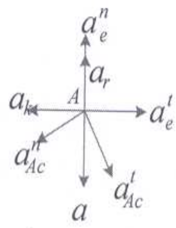
$\therefore M_A = P\times r - \sqrt{3}P\times a + P\times 2l$

二 解





3分



3分

1 以轮 O 为研究对象

$\omega_O = \frac{v_o}{r}, \therefore v_C = 2v_o$ 3分

2 以三角形 ABC 为研究对象，B 点为瞬心

$v_A = v_C = 2v_o, \omega_\Delta = \frac{v_C}{BC} = \frac{2v_o}{2r} = \frac{v_o}{r}$ 3分

3 以 A 为动点，O_2D 为动系

$v_e = \frac{1}{2}v_A = v_o, v_r = \frac{\sqrt{3}}{2}v_A = \sqrt{3}v_o$ 3分

$\therefore \omega_{O2} = \frac{v_e}{O_2A} = \frac{v_o}{2r}$

4 以 O 为基点，求 C 点加速度

$\vec{a}_c = \vec{a}_o + \vec{a}^n_{co} + \vec{a}^t_{co}$

$\because a_O = 0, \therefore \alpha_O = 0, \therefore a^t_{CO} = 0$ 3分

$\vec{a}_c = \vec{a}^n_{co}, a_C = a^n_{CO} = \omega_O^2 r = \frac{v_O^2}{r}$

5 以 C 为基点，求 B 点加速度

$\vec{a}_B = \vec{a}_c + \vec{a}^n_{BC} + \vec{a}^t_{BC}$

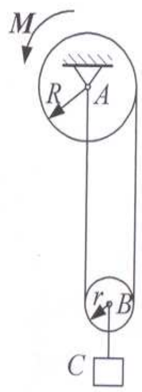
在铅垂方向投影 3分

$a^t_{AC} = 0, \therefore \alpha_\Delta = 0$

三 解

设重物上升 h 距离时。A 轮转动 S 的弧长 $S = R\varphi = 2h, \therefore \varphi = 2h/R$ 5分

外力功：$W = M\varphi - m_2gh - m_3gh = (2\dfrac{M}{R} - m_2g - m_3g)h$



$$\because J_A = \frac{1}{2}m_1R^2, J_B = \frac{1}{2}m_2r^2$$

6分

$$v_B = v_C = v, \omega_B = \frac{v}{r}, \omega_A = \frac{2r}{R}\omega_B = \frac{2v}{R}$$

动能：

$$T_A = \frac{1}{2}J_A\omega_A^2 = m_1v^2$$

$$T_B = \frac{1}{2}m_2v_B^2 + \frac{1}{2}J_B\omega_B^2 = \frac{3}{4}m_2v^2$$

6分

$$T_C = \frac{1}{2}m_3v_C^2 = \frac{1}{2}m_3v^2$$

$$T = T_A + T_B + T_C = (m_1 + \frac{3}{4}m_2 + \frac{1}{2}m_3)v^2$$

由动能定理

$$(m_1 + \frac{3}{4}m_2 + \frac{1}{2}m_3)v^2 = (2\frac{M}{R} - m_2g - m_3g)h$$ 3分

两边对 t 求导 $\dfrac{dh}{dt} = v, \dfrac{dv}{dt} = a$

$$(2m_1 + \frac{3}{2}m_2 + m_3)va = (2\frac{M}{R} - m_2g - m_3g)v$$ 5分

$$a = \frac{2\dfrac{M}{R} - m_2g - m_3g}{2m_1 + \dfrac{3}{2}m_2 + m_3}$$

四 解

1 由动能定理

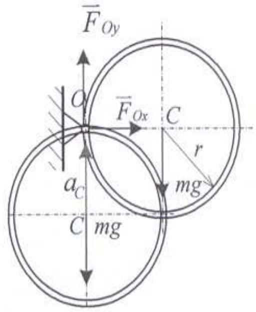
$J_o = J_C + mr^2 = 2mr^2$ 3分

$T_1 = 0, T_2 = \frac{1}{2} J_o \omega^2 = mr^2 \omega^2$ 4分

$W = mgr$

$\therefore mr^2 \omega^2 = mgr$

$\omega^2 = g/r, \omega = \sqrt{g/r}$ 4分



2 由动量矩定理

$J_o \alpha = \sum M_O(\vec{F})$

$J_o \alpha = 0$ 6分

$\alpha = 0$

$a_c^t = 0, a_C = a_c^n = r\omega^2 = g$

3 由质心运动定理

$ma_C = F_{Oy} - mg$ 4分

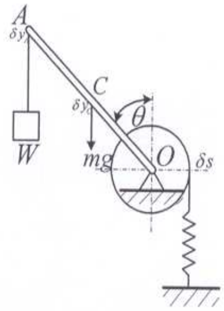
$0 = F_{Ox}$

$\therefore F_{Ox} = 0$

$F_{Oy} = ma_C + mg = 2mg$ 4分

五 解

由虚位移原理



$$W\delta y_A + mg\delta y_C + F\delta s = 0$$ 3分

$$y_A = l\cos\theta, y_C = \frac{l}{2}\cos\theta, s = r\theta$$ 3分

$$\therefore \delta y_A = -l\sin\theta\delta\theta$$ 3分

$$\delta y_C = -\frac{l}{2}\sin\theta\delta\theta$$ 3分

$$\delta s = r\delta\theta$$

$$F = ks = kr\theta$$ 3分

$$\therefore -Wl\sin\theta\delta\theta - mg\frac{l}{2}\sin\theta\delta\theta + kr^2\theta\delta\theta = 0$$ 3分

$$-Wl\sin\theta - mg\frac{l}{2}\sin\theta + kr^2\theta = 0$$ 3分

$$g \approx 10kgm/s^2$$

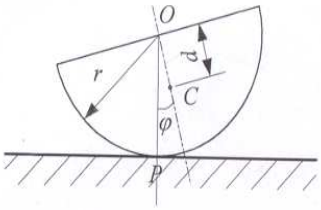
$$\therefore 8\sin\theta = 5\theta$$ 4分

六　解

应用拉格朗日方程，一个自由度 φ　3分

动能：

$\frac{1}{2}J_P\dot{\varphi}^2$　3分



$J_P = J_C + m\overline{PC}^2 = m\rho^2 + m(r^2 + d^2 - 2rd\cos\varphi)$

$\therefore T = (\frac{1}{2}m\rho^2 + \frac{1}{2}mr^2 + \frac{1}{2}md^2 - mrd\cos\varphi)\dot{\varphi}^2$　5分

势能：

$V = mgd(1-\cos\varphi)$　5分

拉格朗日函数

$L = T - V$

$= (\frac{1}{2}m\rho^2 + \frac{1}{2}mr^2 + \frac{1}{2}md^2 - mrd\cos\varphi)\dot{\varphi}^2 - mgd(1-\cos\varphi)$　2分

$\frac{\partial L}{\partial \dot{\varphi}} = (m\rho^2 + mr^2 + md^2 - 2mrd\cos\varphi)\dot{\varphi}$　3分

$\frac{d}{dt}(\frac{\partial L}{\partial \dot{\varphi}}) = (m\rho^2 + mr^2 + md^2 - 2mrd\cos\varphi)\ddot{\varphi} + 2mrd\sin\varphi\dot{\varphi}^2$　3分

$\frac{\partial L}{\partial \varphi} = mrd\sin\varphi\dot{\varphi}^2 - mgd\sin\varphi$　3分

$\therefore (m\rho^2 + mr^2 + md^2 - 2mrd\cos\varphi)\ddot{\varphi} + 2mrd\sin\varphi\dot{\varphi}^2 - mrd\sin\varphi\dot{\varphi}^2 + mgd\sin\varphi = 0$

$(m\rho^2 + mr^2 + md^2 - 2mrd\cos\varphi)\ddot{\varphi} + mrd\sin\varphi\dot{\varphi}^2 + mgd\sin\varphi = 0$

微振动 $\sin\varphi \approx \varphi . \cos\varphi \approx 1, \varphi\dot{\varphi}^2 \approx 0$

3分

$(m\rho^2 + mr^2 + md^2 - 2mrd)\ddot{\varphi} + mrd\varphi\dot{\varphi}^2 + mgd\varphi = 0$

$\therefore (m\rho^2 + mr^2 + md^2 - 2mrd)\ddot{\varphi} + mgd\varphi = 0$