一、求下列周期信号的周期

1、

$$(a\cos 2t)^2 = a^2\cos^2 2t = a^2\frac{1+\cos 4t}{2} = a^2\frac{1+\cos(4t+4T)}{2}$$

$$4T = 2\pi \qquad T = \frac{\pi}{2}$$

2、

$$(\sin 2t + 2\sin 5t)^2 = \sin^2 2t + 4\sin 2t\sin 5t + 4\sin^2 5t$$

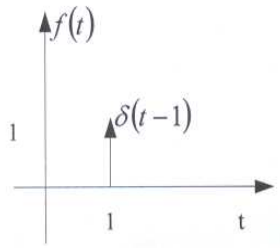
$$= \frac{1-\cos 4t}{2} + 4\frac{\sin(5-2)t - \sin(5+2)t}{2} + \frac{1-\cos 10t}{2}$$

$$\omega_1 = 3 \quad \omega_2 = 4 \quad \omega_3 = 7 \quad \omega_4 = 10 \quad m_1 = 3$$

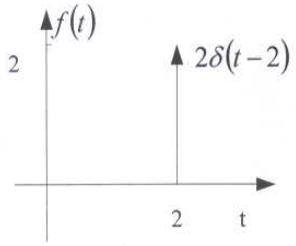
$$T = m_1\frac{2\pi}{3} = 2\pi$$

二、画波形：

1、$f(t) = 2t\delta(2t-2)$



2、$f(t) = 2\varepsilon(t)\delta(t-2)$



三、计算

1、$f(t) = e^t\varepsilon(t)\delta(t-2) = e^2\delta(t-2)$

2、$\int_{-\infty}^{3}\delta(t)\frac{\sin(3t)}{t}dt = 0$

四、富氏变换

$$f(t)=e^{-t}\sin(t)\varepsilon(t)$$

$$\varepsilon(t)\Rightarrow \pi\delta(\omega)+\frac{1}{j\omega}$$

1、$e^{-t}\varepsilon(t)\Rightarrow e^{j*jt}\varepsilon(t)\Rightarrow \pi\delta(\omega-j)+\frac{1}{j(\omega-j)}=0+\frac{1}{j\omega+1}=\frac{1}{j\omega+1}$

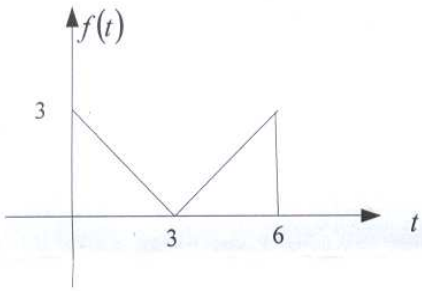
$$e^{-t}\sin(t)\varepsilon(t)\Rightarrow \frac{1}{2}j\left[\frac{1}{j(\omega+1)+1}-\frac{1}{j(\omega-1)+1}\right]=\frac{1}{2}j\left[\frac{1}{(j\omega+1)+j}-\frac{1}{(j\omega+1)-j}\right]$$

$$=\frac{1}{2}j\frac{(j\omega+1)-j-(j\omega+1)-j}{(j\omega+1)^2+1}=\frac{1}{(j\omega+1)^2+1}$$

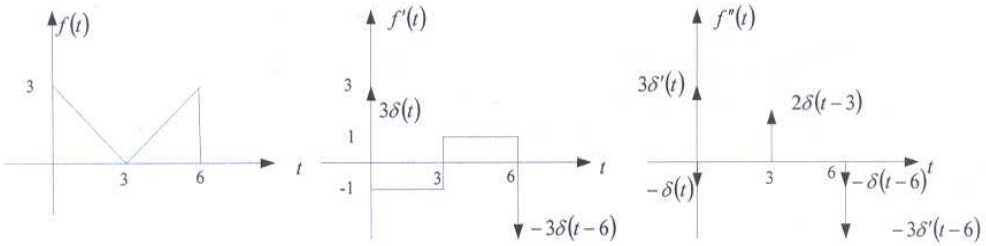
2、$g_2(t-1)$

$$g_2(t)\Rightarrow 2\frac{\sin\omega}{\omega}\qquad g_2(t-1)\Rightarrow 2\frac{\sin\omega}{\omega}e^{-j\omega}$$

3、



解答



$$f''(t)=3\delta'(t)-\delta(t)+2\delta(t-3)-\delta(t-6)-3\delta'(t-6)$$

$$F(j\omega)=\frac{1}{(j\omega)^2}\left[j3\omega-1+2e^{-j3\omega}-e^{-j6\omega}-j3\omega e^{-j6\omega}\right]$$

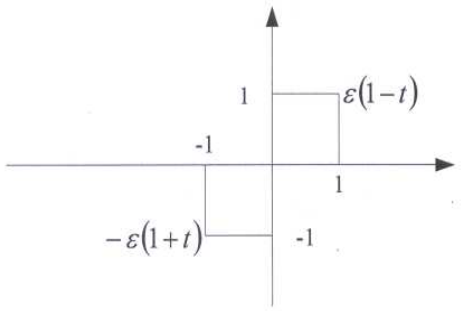
4、$f(t)=\varepsilon(1-|t|)\mathrm{sgn}(t)$

$t \le 0, \quad f(t) = \varepsilon(1+t)(-1) = -\varepsilon(t+1)$

$t \ge 0, \quad f(t) = \varepsilon(1-t)$

$\because \quad f(t) = g_1(t-0.5) - g_1(t+0.5)$

$$\Rightarrow \frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}}\left(e^{-j0.5\omega} - e^{j0.5\omega}\right) = -j2Sa\left(\frac{\omega}{2}\right)\left(\sin\frac{\omega}{2}\right) = -j4\frac{\left(\sin\frac{\omega}{2}\right)^2}{\omega}$$



五、已知 $f(t) = te^{-2t}\varepsilon(t)$ 的傅氏变换为 $F(j\omega)$，求对应下列频谱的时间函数。

1)、$F(j2\omega)$

$\because \quad f\left(\frac{t}{2}\right) \Leftrightarrow 2F(j2\omega)$

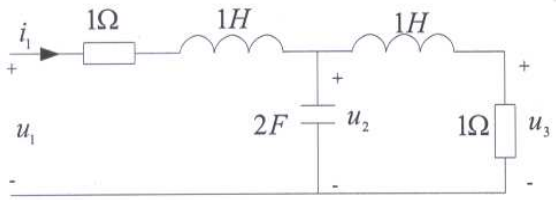
$\therefore \quad F(j2\omega) \Leftrightarrow \frac{1}{2}f\left(\frac{t}{2}\right) = \frac{1}{4}te^{-t}\varepsilon(t)$

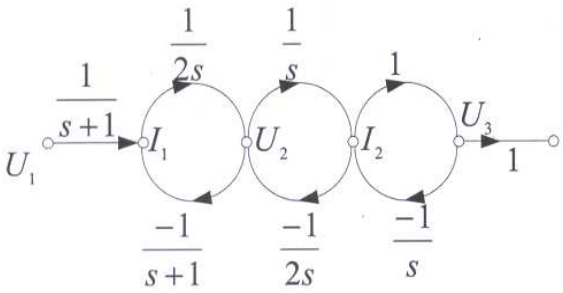
2)、$F(\omega-1) + F(\omega+1)$

由调制定理 $\qquad f(t)\cos t \Leftrightarrow 0.5[F(\omega-1) + F(\omega+1)]$

所以 $\quad F(\omega-1) + F(\omega+1) \Leftrightarrow 2f(t)\cos t = 2te^{-2t}\cos t \cdot \varepsilon(t)$

六、画出下图电路的信号流图，求其输入阻抗函数 $Z(s)=\frac{U_1(s)}{I_1(s)}$ 和电压比函数 $H(s)=\frac{U_3(s)}{U_1(s)}$





$$I_1=\frac{1}{1+s}(U_1-U_2)$$

$$U_2=\frac{1}{2s}(I_1-I_2)$$

$$I_1=\frac{1}{s}(U_2-U_3)$$

$$U_3=I_2$$

$$-\sum L_i=\frac{1}{2s(1+s)}+\frac{1}{2s^2}+\frac{1}{s}$$

$$\sum L_m L_n=\frac{-1}{2s(1+s)}\cdot\frac{-1}{s}=\frac{1}{2s^2(1+s)}$$

前向增益：$\frac{1}{2s}\cdot\frac{1}{s}\cdot\frac{1}{1+s}=\frac{1}{2s^2(1+s)}$

$$\Delta=1+\frac{1}{2s(1+s)}+\frac{1}{2s^2}+\frac{1}{s}+\frac{1}{2s^2(1+s)}=\frac{2(s^3+2s^2+2s+1)}{2s^2(1+s)}$$

$$\therefore\frac{U_3}{U_1}=\frac{\frac{1}{2s^2(1+s)}}{1+\frac{1}{2s(1+s)}+\frac{1}{2s^2}+\frac{1}{s}+\frac{1}{2s^2(1+s)}}=\frac{1}{2(s^3+2s^2+2s+1)}$$

又：导纳：$\frac{I_1}{U_1}$；前向增益：$P_1=\frac{1}{s+1}$

$$\Delta_1=1+\frac{1}{2s^2}+\frac{1}{s}=\frac{2s^2+2s+1}{2s^2}$$

$$P_1\Delta_1=\frac{1}{1+s}\cdot\frac{2s^2+2s+1}{2s^2}=\frac{2s^2+2s+1}{2s^2(1+s)}$$

$$Y_i=\frac{I_1}{U_1}=\frac{P_1\Delta_1}{\Delta}=\frac{2s^2+2s+1}{2(s^3+2s^2+2s+1)}$$

$$Z_i=\frac{2(s^3+2s^2+2s+1)}{2s^2+2s+1}$$