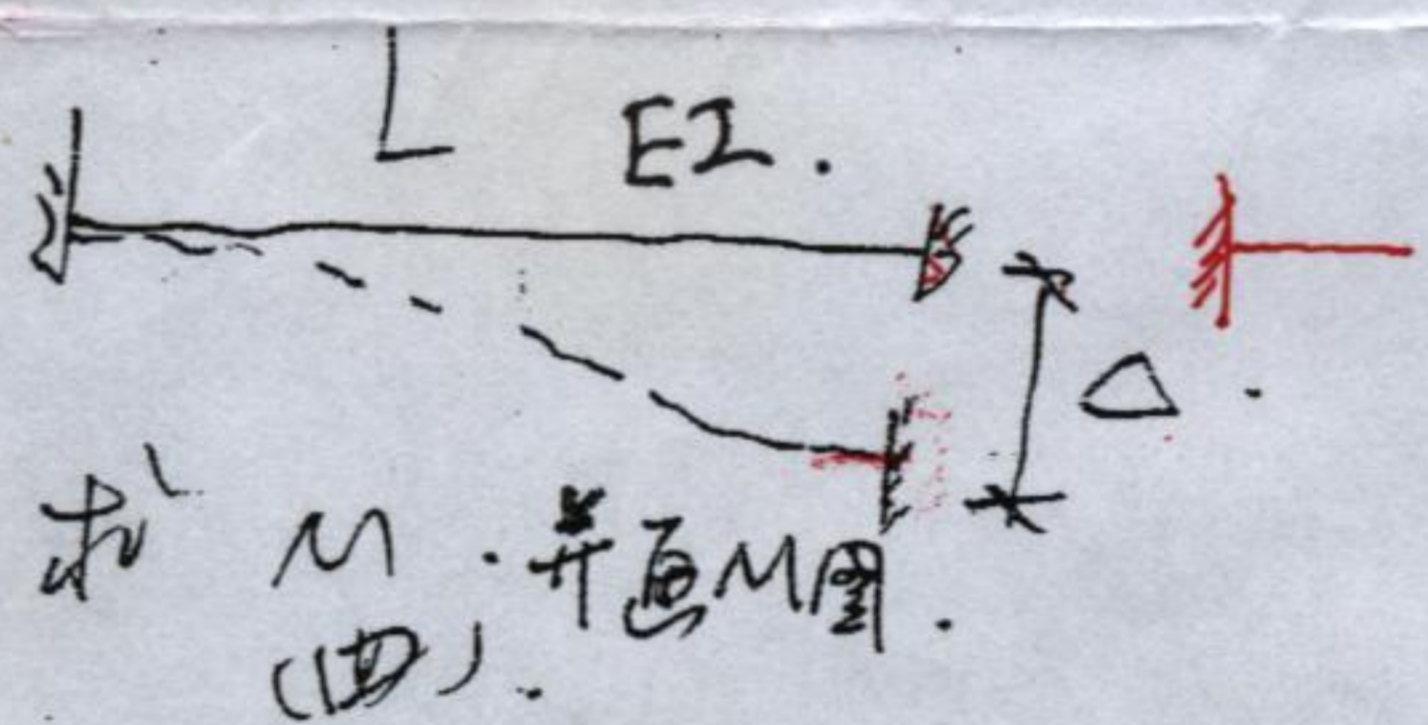
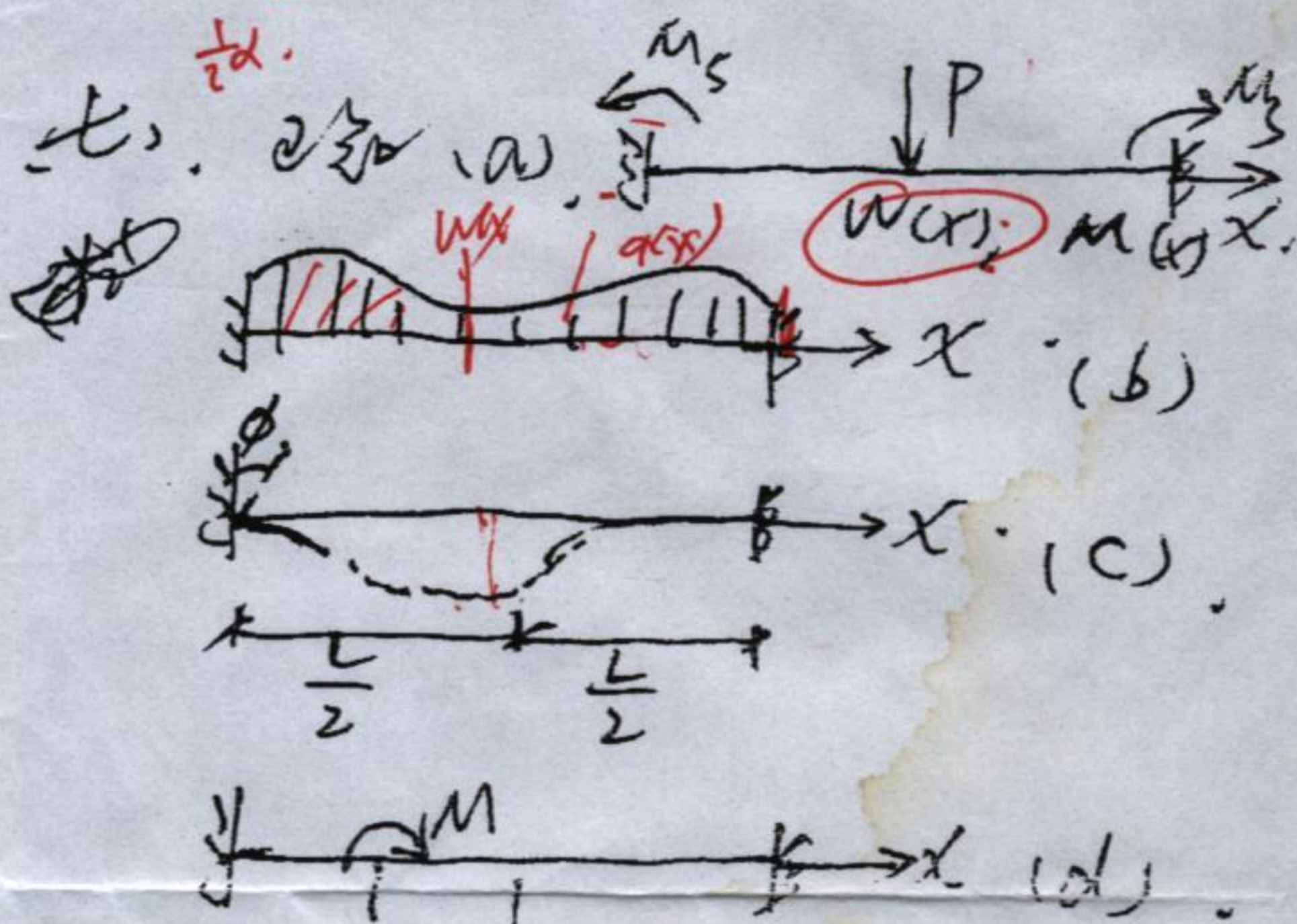
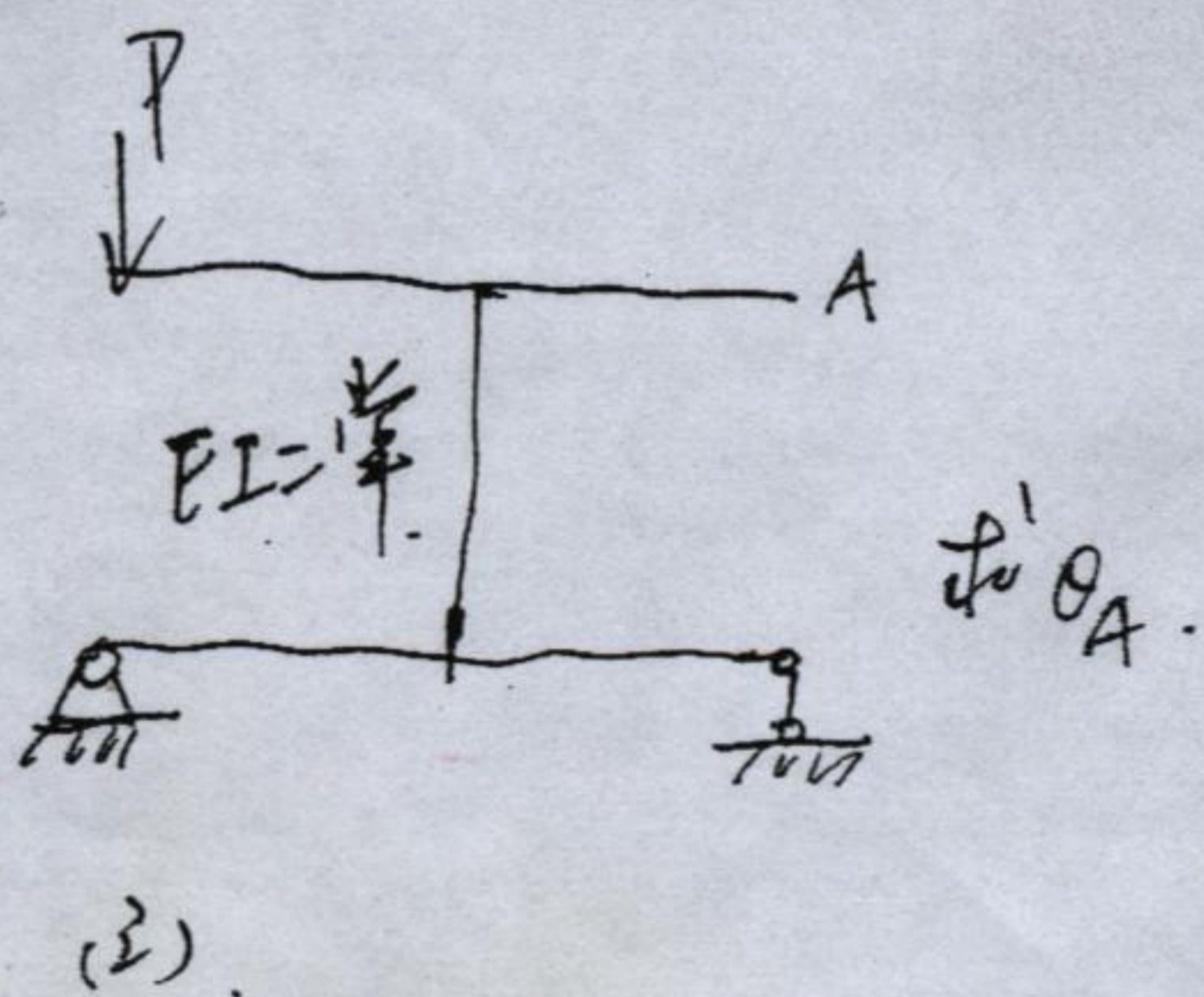
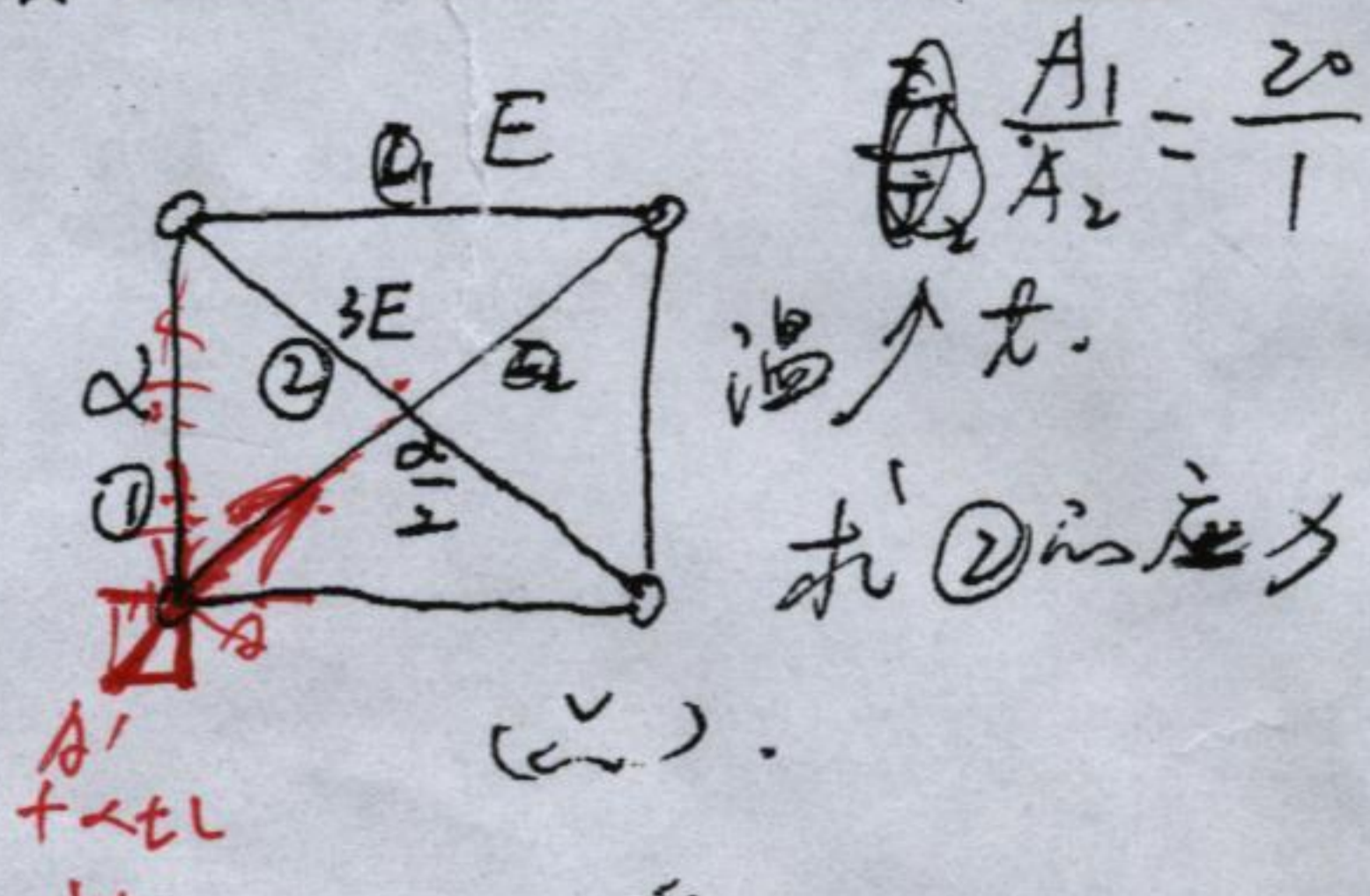
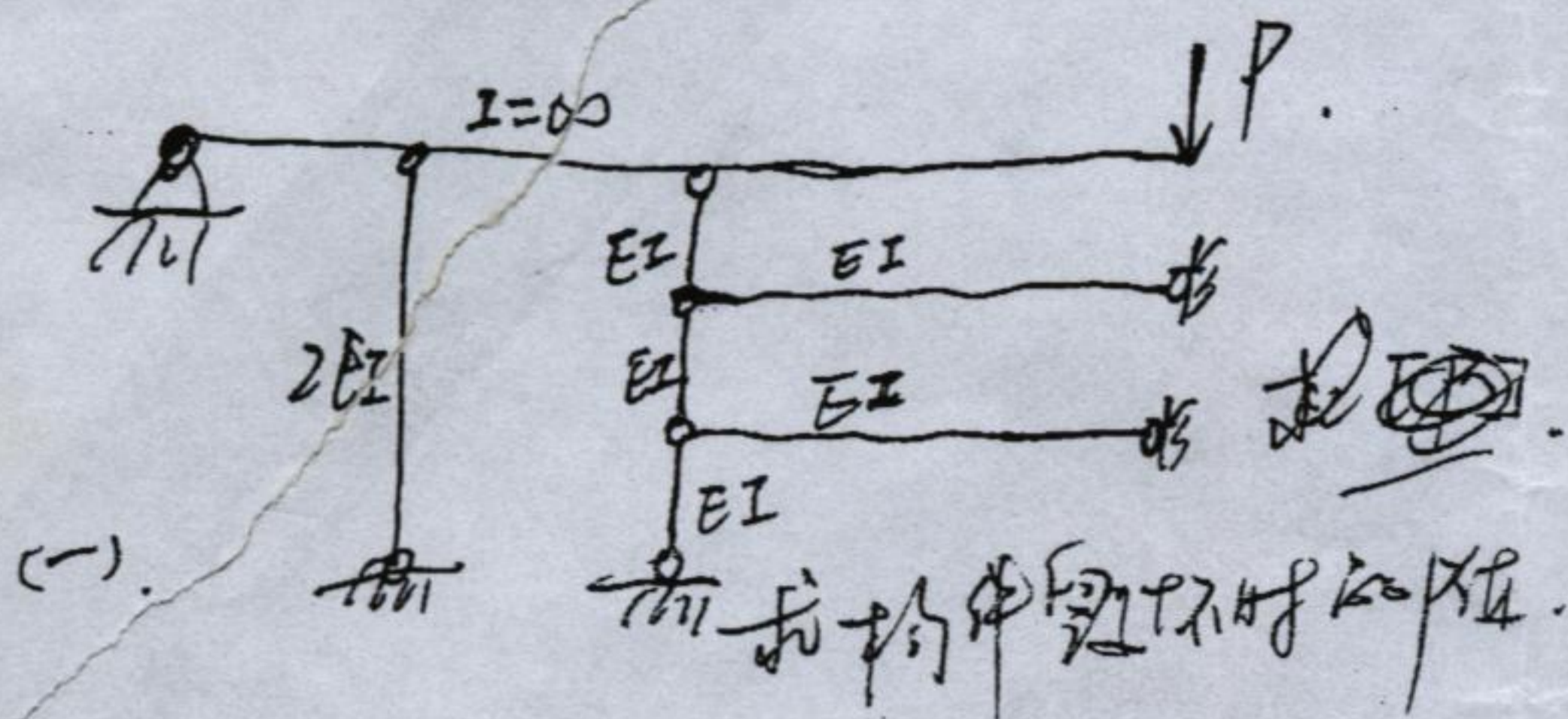
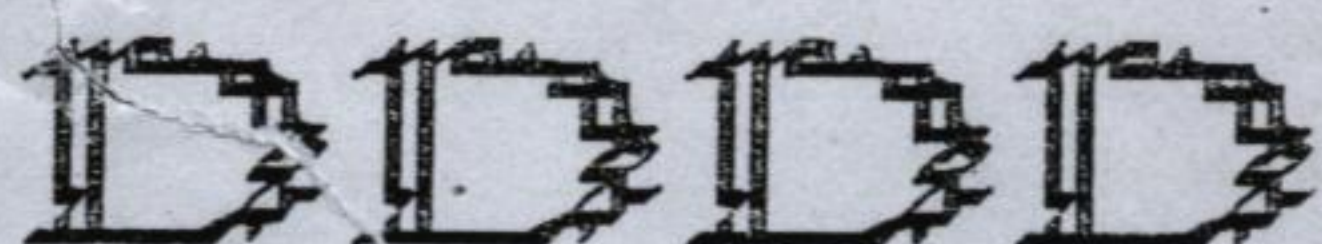
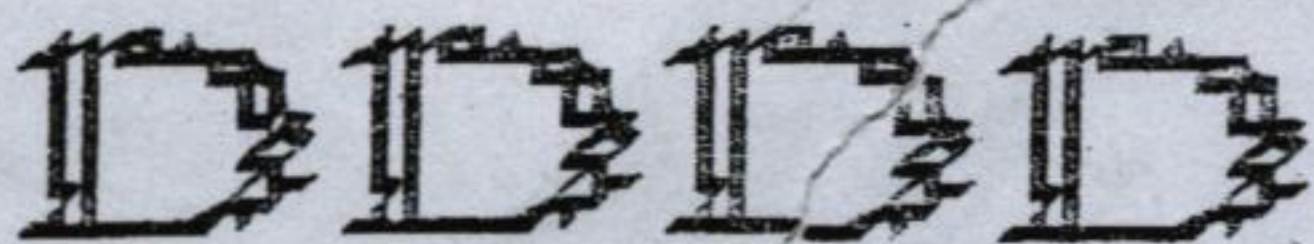


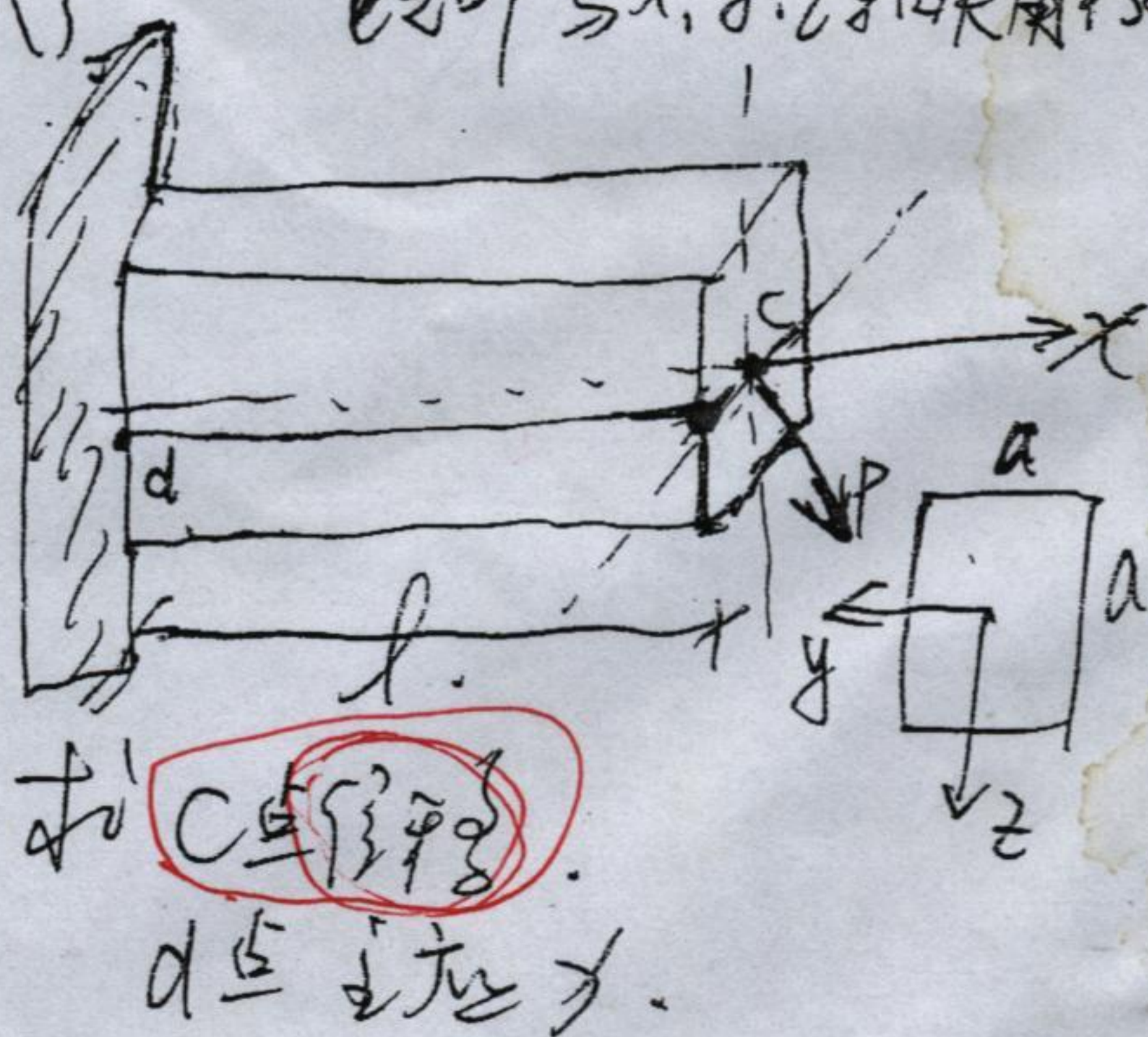
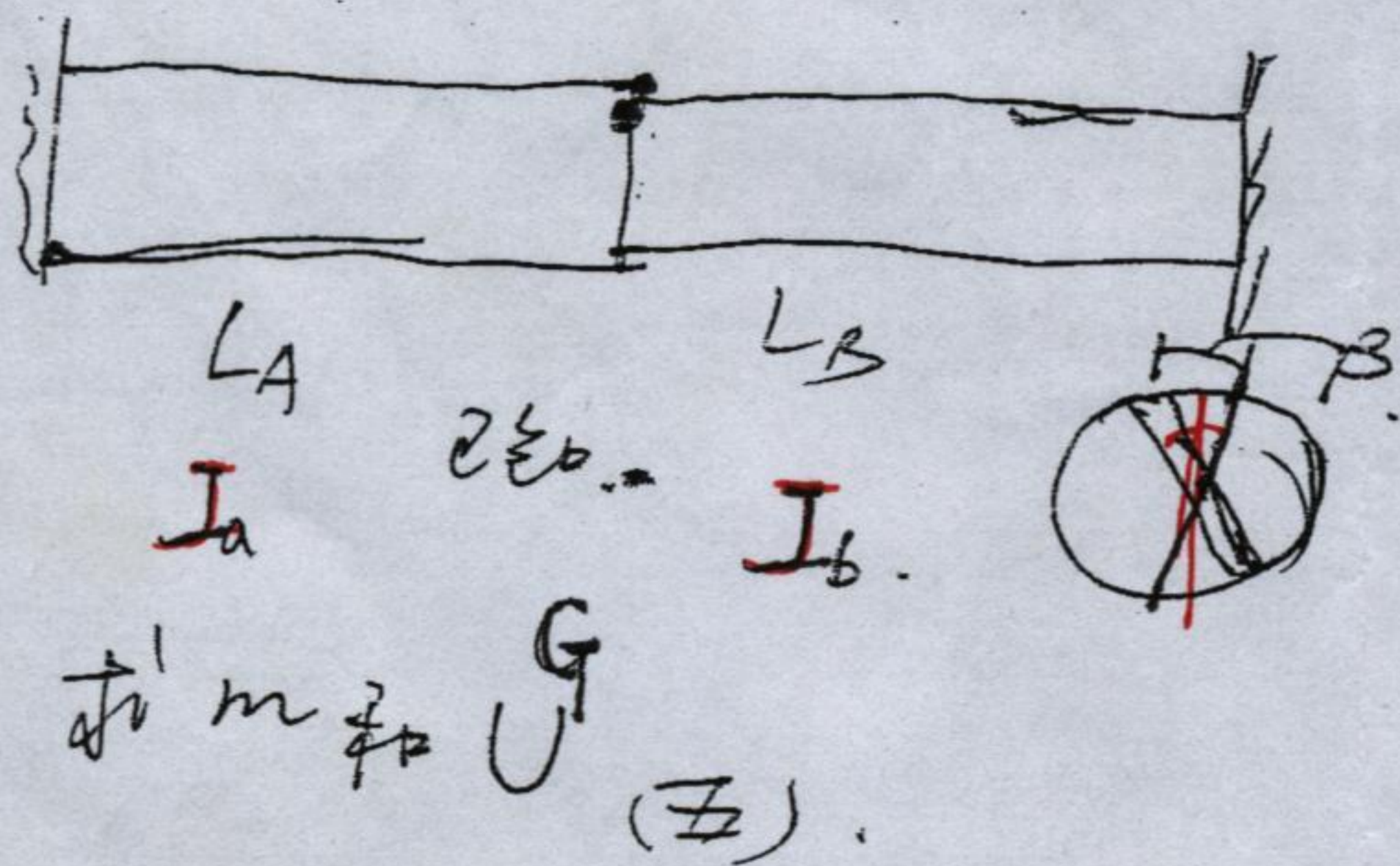
2004年土木工程材料力学

同济大学
研究生考试草稿纸

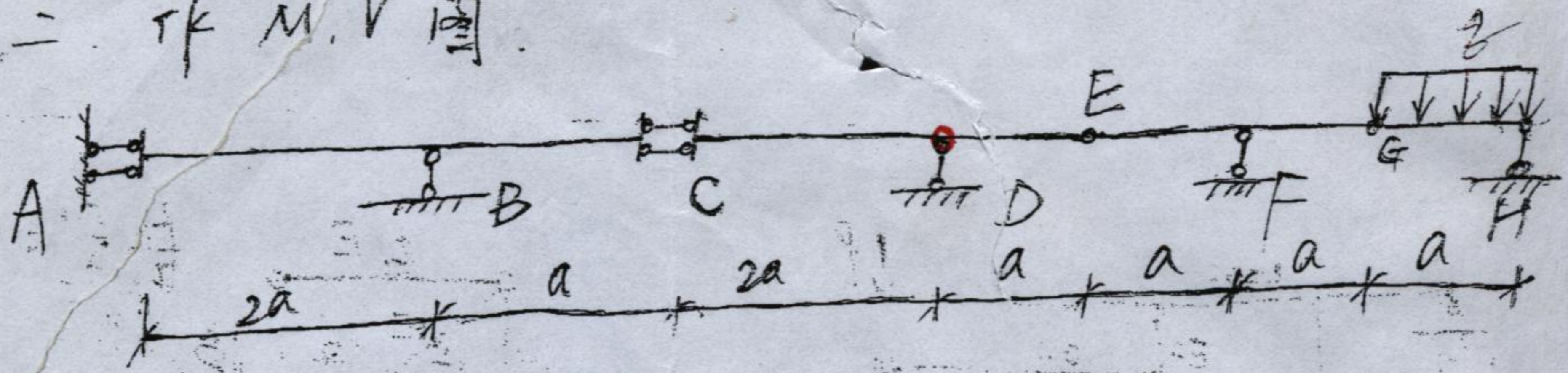


求 (b), (c), (d) 中 C 的轴力。

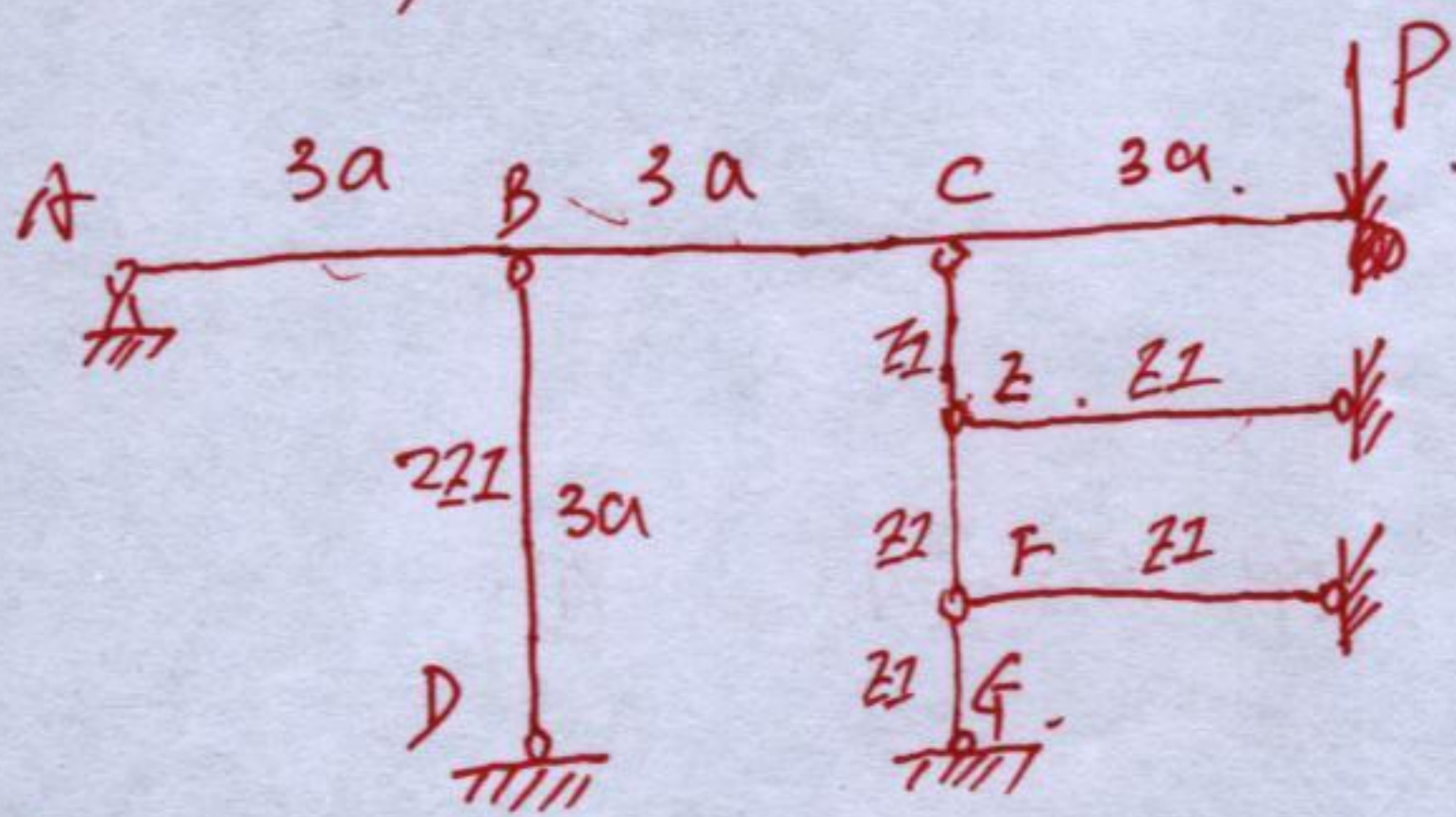
求 P 与 x, y, z 轴夹角。



二. τ_k M.V 图.



(一) 解:



解: 由平衡方程. $\sum M_B = 0$. $P \cdot 9a - N_{BD} \cdot 3a - N_{CE} \cdot 6a = 0 \dots (1)$

由变形协调方程: $\Delta l_{BD} = \frac{1}{2} \Delta l_{CG} \dots (2)$

物理方程为: $\Delta l_{BD} = \frac{N_{BD} \cdot 3a}{2ZI}$

$\Delta l_{CG} = 3 \cdot \frac{N_{CE} \cdot a}{ZI}$

把物理方程代入协调方程有.

$\frac{N_{BD} \cdot 3a}{2ZI} = \frac{1}{2} \cdot \frac{3N_{CE} \cdot a}{ZI} \Rightarrow N_{BD} = N_{CE} \dots (3)$

② 代入①得: $N_{BD} = P$
 $N_{CE} = P$

由稳定条件有: $P_{cr}^{BD} = \frac{2\pi^2 ZI}{(3a)^2}$

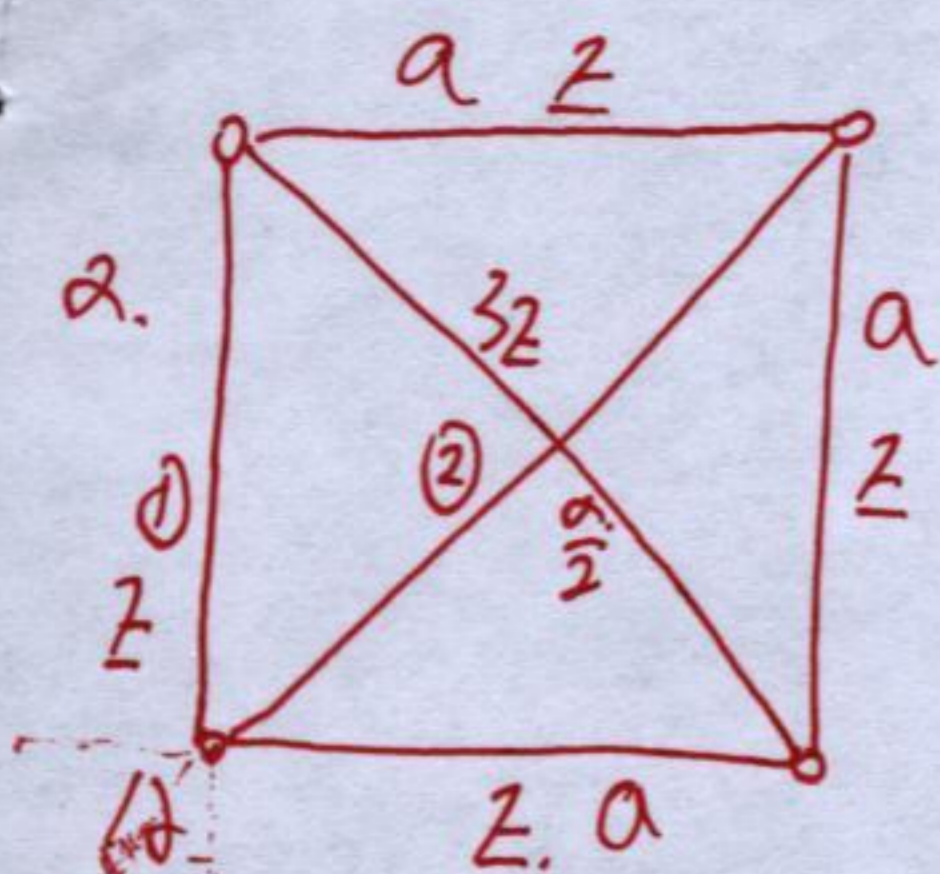
$P_{cr}^{CE} = \frac{\pi^2 ZI}{a^2}$

即有 $\frac{P_{cr}^{BD}}{N_{BD}} \leq n_w \Rightarrow P_1 \leq \dots$
 $\frac{P_{cr}^{CE}}{N_{CE}} \leq n_w \Rightarrow P_2 \leq \dots$
 $\Rightarrow P = \max\{P_1, P_2\}$

因为当杆BD破坏时, 整个构件并未破坏, 当杆CE, EF, FG破坏时构件破坏, 所以 $P = \max\{P_1, P_2\}$

$\frac{a^4}{12} \cdot \frac{\sigma}{2}$

7



平衡方程: $N_1 = N_2 \cos 45^\circ$ 即 $N_1 = \frac{\sqrt{2}}{2} N_2 \dots \textcircled{1}$

变形协调条件:

$$\alpha \cdot t \cdot a - \frac{N_1 a}{EA_1} = \left(\frac{1}{2} \alpha t \cdot \sqrt{2} a + \frac{N_2 \cdot \sqrt{2} a}{3EA} \right) \cdot \frac{\sqrt{2}}{2}$$

$$\text{即: } \frac{1}{2} \alpha t a = \frac{N_1 a}{EA_1} + \frac{N_2 a}{3EA_2}$$

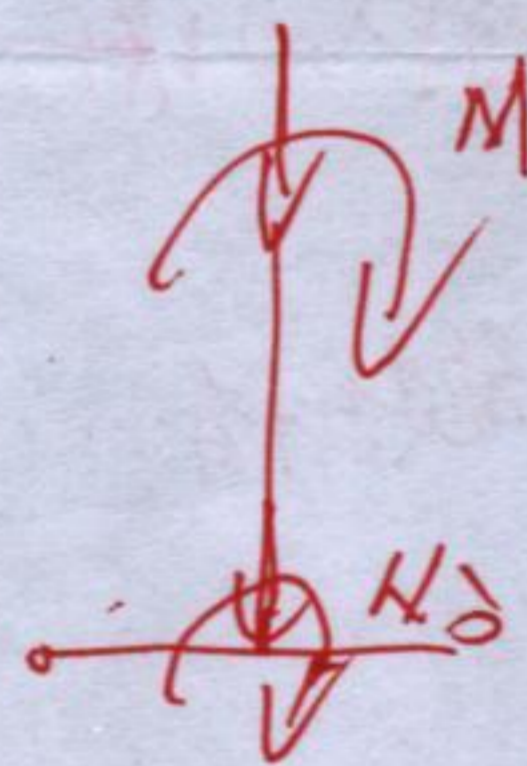
$$\because \frac{A_1}{A_2} = \frac{20}{1}$$

$$\text{又有 } \frac{1}{2} \alpha t = \frac{N_1}{E \cdot 20A_2} + \frac{N_2}{3EA_2} \dots \textcircled{2}$$

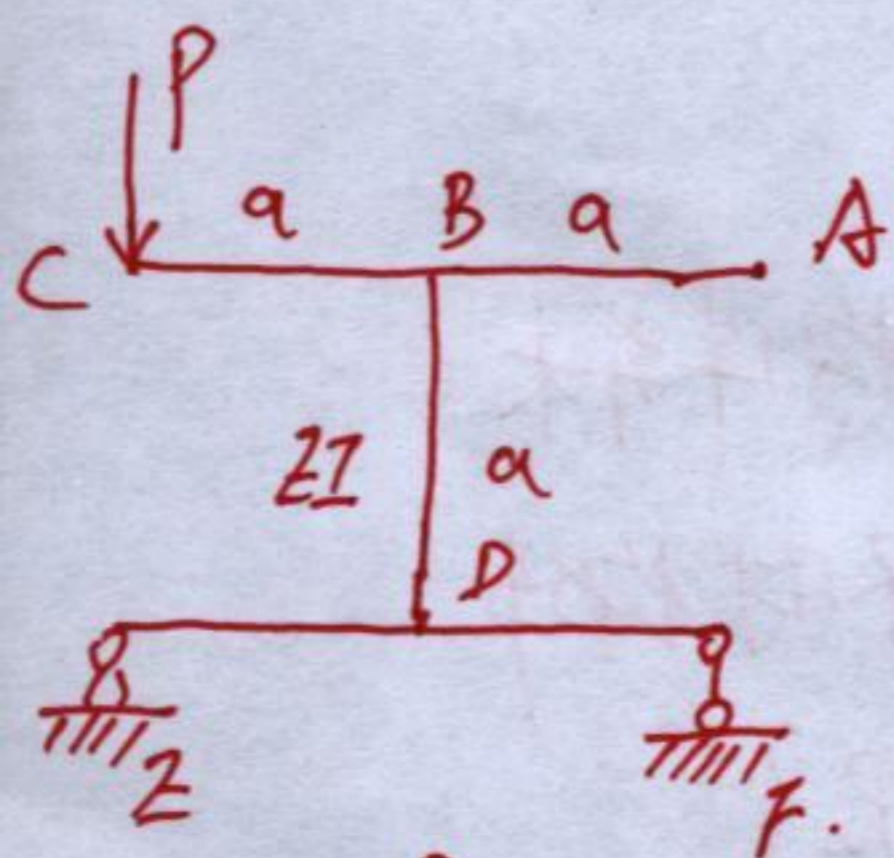
由①②解得:

$$N_2 = \frac{\frac{1}{2} \alpha t EA_2}{\frac{\sqrt{2}}{40} + \frac{1}{32}} = \frac{60 \alpha t EA_2}{3\sqrt{2} + 40}$$

$$\therefore G_2 = \frac{N_2}{A_2} = \frac{60 \alpha t E}{3\sqrt{2} + 40}$$



(三)

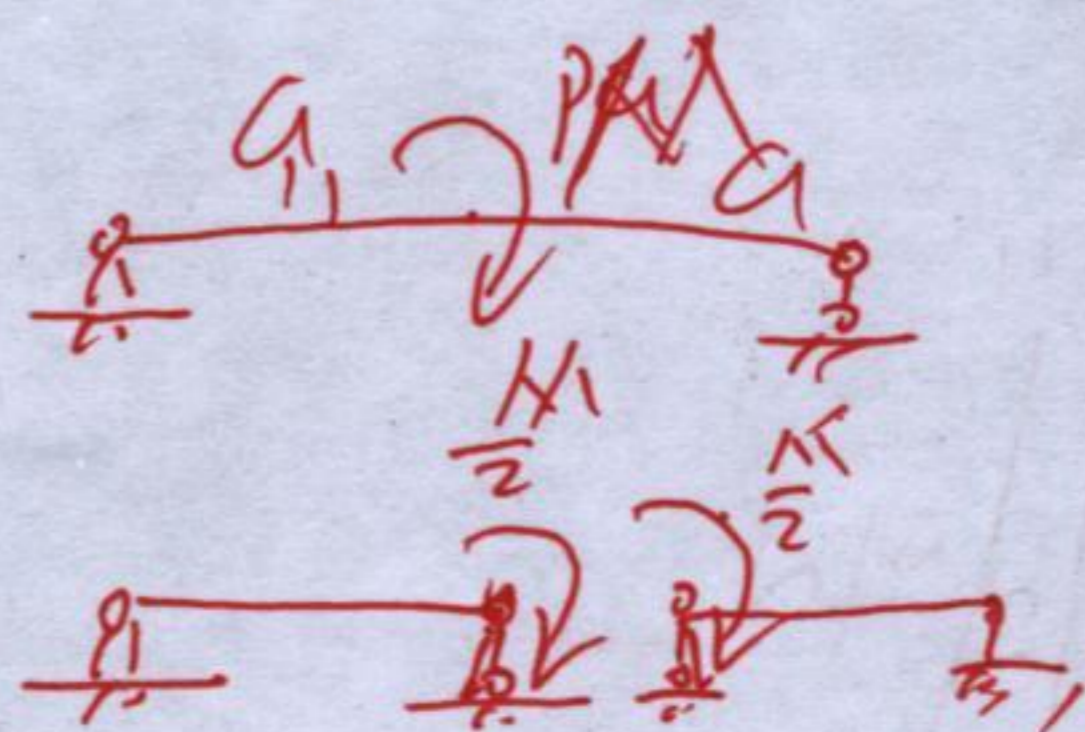


解: $\theta_A = \theta_{A1} + \theta_{A2}$

$$\theta_{A1} = \frac{(Pa)a}{EI} = \frac{Pa^2}{EI}$$

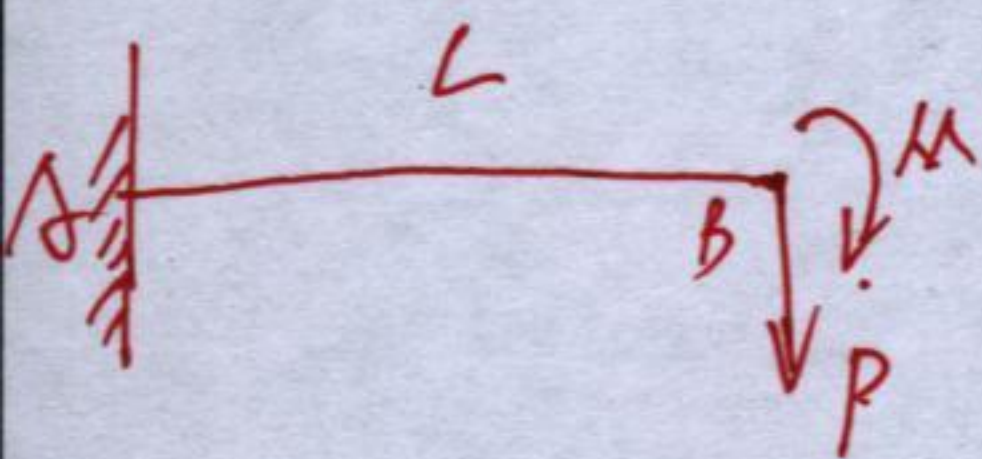
$$\theta_{A2} = \frac{\frac{Pa}{2} a}{3EI} = \frac{Pa^2}{6EI}$$

$$\therefore \theta_A = \frac{Pa^2}{EI} + \frac{Pa^2}{6EI} = \frac{7Pa^2}{6EI}$$



$$\frac{\frac{Pa}{2} \cdot a}{3EI} = \frac{Pa^2}{6EI}$$

(三)



在 B 点去除约束以 P 和 M 取代。

由功可知:
$$\delta_B = \frac{ML^2}{2EI} + \frac{P \cdot L^3}{3EI}$$

$$\theta_B = \frac{PL^2}{2EI} + \frac{ML}{EI}$$

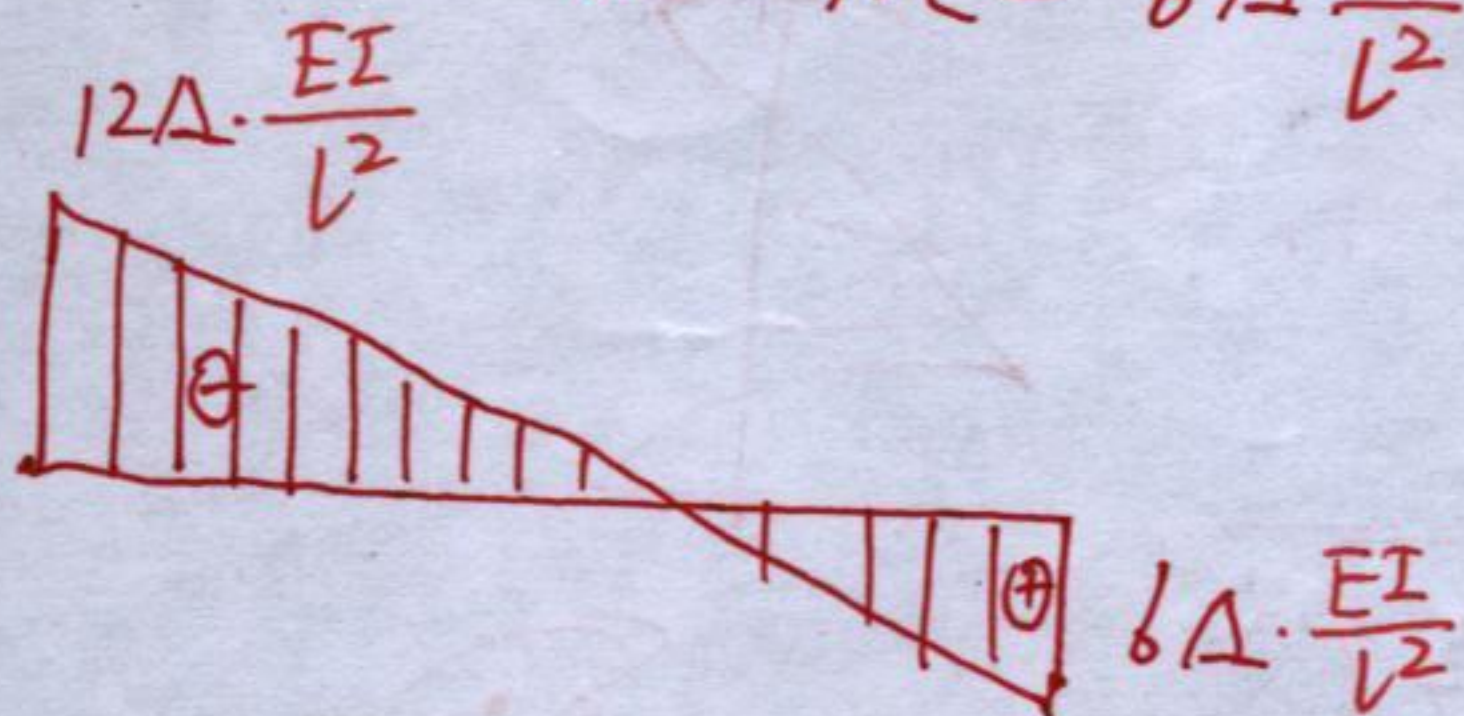
即: $\delta_B = \Delta$

$\theta_B = 0$

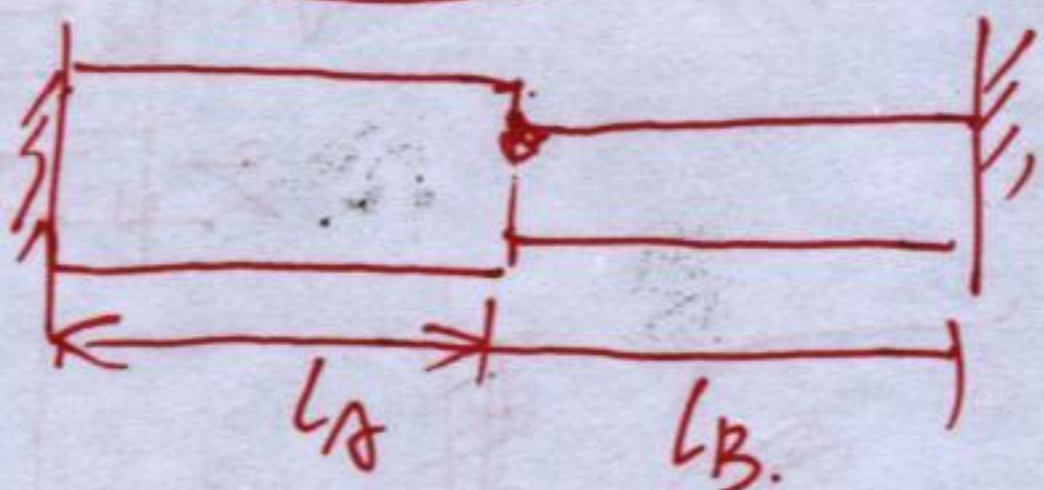
又有:
$$\begin{cases} \frac{ML^2}{2EI} + \frac{PL^3}{3EI} = \Delta \\ \frac{PL^2}{2} + M = 0 \end{cases} \Rightarrow \begin{cases} M = -6\Delta \cdot \frac{EI}{L^2} = -\frac{6\Delta EI}{L^2} \\ P = \frac{12\Delta}{L} \cdot \frac{EI}{L^2} = \frac{12\Delta EI}{L^3} \end{cases}$$

故有: $M_A = -P \cdot L = -12\Delta \cdot \frac{EI}{L^2}$

$M_B = -M = 6\Delta \cdot \frac{EI}{L^2}$



(四)



$$U = \frac{T_A^2 L_A}{2G_A J_A} + \frac{T_B^2 L_B}{2G_B J_B}$$

= ...

解: 设其扭矩分别为 T_A 和 T_B

则有 $\frac{T_A L_A}{G_A J_A} + \frac{T_B L_B}{G_B J_B} = \beta \leftarrow \varphi_A + \varphi_B = \beta$

即 $T_A = T_B$

故有: $T_A = T_B = \frac{\beta}{\frac{L_A}{G_A J_A} + \frac{L_B}{G_B J_B}}$



(1) 解: 把 \$P\$ 分解为 \$P_x, P_y, P_z\$.

$$P_x = P_y = P_z = P/\sqrt{3}$$

故有: 轴力 \$P_x = \frac{P}{\sqrt{3}}\$

弯矩: $M_z = P_y L = \frac{PL}{\sqrt{3}}$

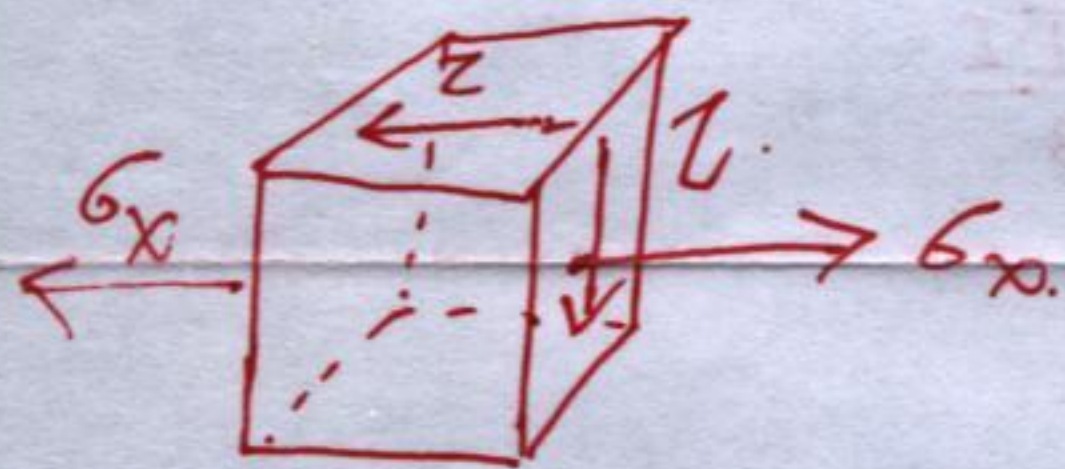
$$M_y = P_z L = \frac{PL}{\sqrt{3}}$$

故有 $\delta_x = \frac{P_x \cdot L}{E \cdot A} = \frac{PL}{3EA^2}$

$$\delta_y = \frac{P_y \cdot l^3}{3EI} = \frac{P}{\sqrt{3}} \cdot \frac{l^3}{3E \cdot \frac{a^4}{12}} = \frac{4Pl^3}{\sqrt{3}EA^4}$$

$$\delta_z = \frac{4Pl^3}{\sqrt{3}EA^4}$$

$$\delta = \sqrt{\delta_x^2 + \delta_y^2 + \delta_z^2}$$



$$\sigma_x = \frac{P_x}{A} + \frac{M_z}{W} = \frac{P}{\sqrt{3}a^2} - \frac{\frac{PL}{\sqrt{3}}}{\frac{ab^3}{6}} = \frac{P}{\sqrt{3}a^2} - \frac{6PL}{\sqrt{3}ab^3}$$

$$\tau = \frac{3Q}{2I} = \frac{3}{2} \cdot \frac{P}{\sqrt{3}a^2} = \frac{\sqrt{3}P}{2a^2}$$

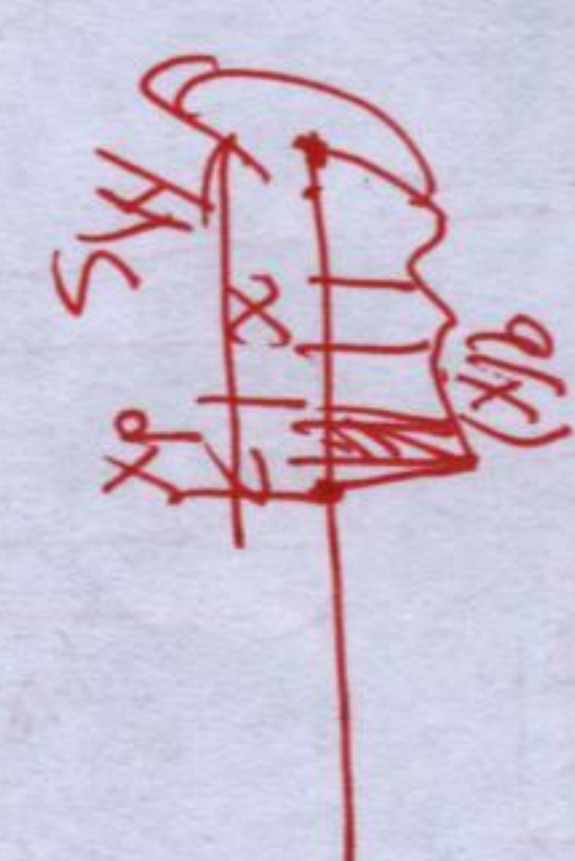
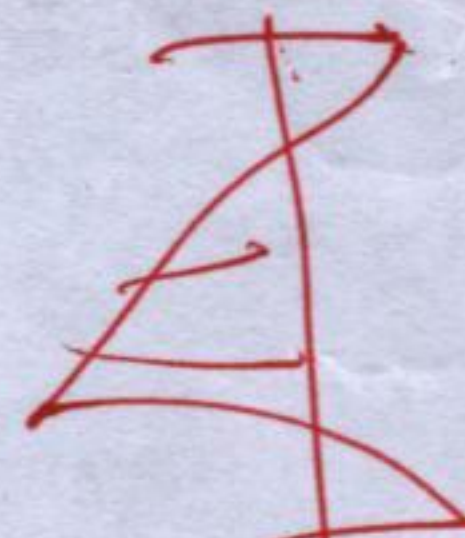
故有:

$$\sigma_{\max/\min} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

故有 $\sigma_1 =$

$\sigma_2 =$

$\sigma_3 =$



$$\int_0^b \sigma(x) dx$$

(七) 由于题可能未写清楚, 此题属于结构力学范围, 但此题应用图乘法解最简单

