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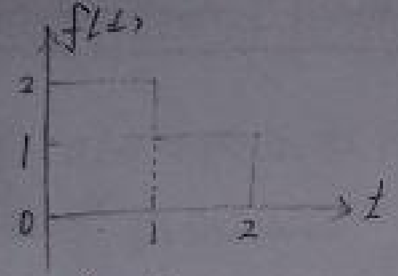
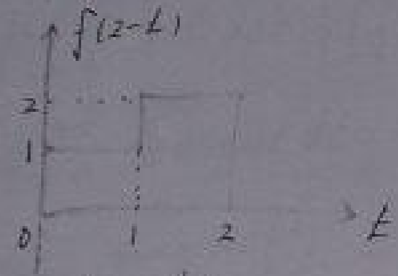
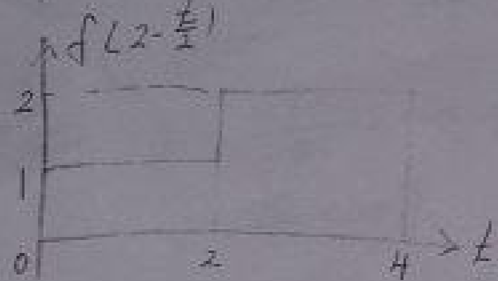
一、

1.







$f(-t) \xleftrightarrow{F} F(-j\omega)$

∴ $F_1(j\omega) = e^{-j2\omega}F(-j\omega)$

$F_1(j\omega) = e^{-j2\omega}F(-j\omega)$

故 $f(4-t) \Leftrightarrow e^{-j4\omega}F(-j\omega)$

∴ $F_2(j\omega) = \frac{1}{\frac{1}{2}} e^{-j\frac{4\omega}{\frac{1}{2}}} F(-j2\omega)$

$= 2e^{-j8\omega}F(-j2\omega)$

2. 由题意: $g(t) \leftrightarrow G(j\omega)$

$g(\frac{t-b}{a}) \Leftrightarrow \frac{1}{|\frac{1}{a}|} e^{-jb\omega} G(\frac{j\omega}{a})$

$= |a| e^{-jb\omega} G(\frac{j\omega}{\frac{1}{a}})$

卷积定理

∴ $R(j\omega) = F(j\omega)\cdot|a|e^{-jb\omega}G(ja\omega)$

$= |a|e^{-jb\omega}F(j\omega)G(ja\omega)$

3. $r(t) = \int_0^{t+a} f(\tau)d\tau - \int_0^t f(\tau)d\tau$

$= f(t) * u(t+a) - f(t)*u(t)$

$R(s) = F(s)\cdot\frac{e^{sa}}{s} - F(s)\frac{1}{s}$

$= \frac{F(s)}{s}(e^{sa}-1)$ ∵

4.

$Y(z) = x(0) + [x(0)+x(1)]z^{-1} + [x(0)+x(1)+x(2)]z^{-2} + [x(0)+x(1)+x(2)+x(3)]z^{-3} + \cdots$

$z^{-1}Y(z) = x(0)z^{-1} + [x(0)+x(1)]z^{-2} + [x(0)+x(1)+x(2)]z^{-3} + \cdots$

$Y(z)(1-z^{-1}) = x(0) + x(1)z^{-1} + x(2)z^{-2} + \cdots = X(z)$

$Y(z) = \frac{X(z)}{1-z^{-1}}$

二、由题意:

$r(t) = [e(t)*h_1(t) - e(t)*h_2(t)]*h_3(t)$

$*[h_1(t) - h_2(t)]*h_2(t)$

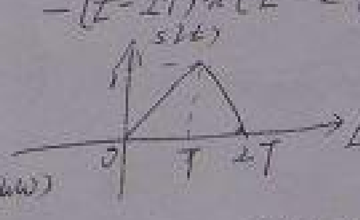
$= e(t)*[h_1(t) - h_2(t)]*h_3(t)$

$*[h_1(t) - h_2(t)]*h_3(t)$

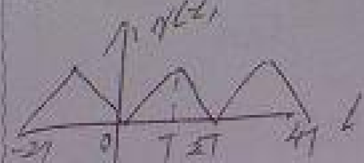
$= e(t)*[tu(t) - 2(t-T)u(t-T) + (t-2T)u(t-2T)]$

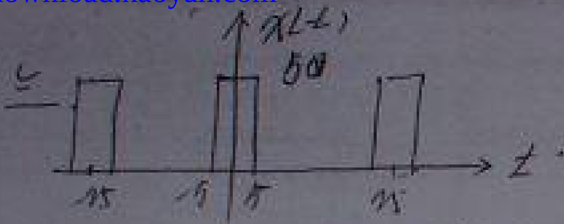
设 $s(t) = tu(t) - 2(t-T)u(t-T) + (t-2T)u(t-2T)$

$= tu(t) - (t-T)u(t-T) - [(t-T)u(t-T) - (t-2T)u(t-2T)]$ 波形如下



∴ $r(t) = e(t)*s(t) = \sum_{n=-\infty}^{\infty} s(t-2nT)$





由题：x(t)为偶函数所以可设 x(t)

$= a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t \qquad \omega = \frac{2\pi}{T}$

$a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\,dt = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} E\,dt = \frac{E\tau}{T}$

$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\cos n\omega t\,dt = \frac{2}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} E\cos n\omega t\,dt$

$= \frac{2E}{Tn\omega}\sin n\omega t\Big|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} = \frac{2E}{Tn\omega}\cdot 2\sin\frac{n\omega\tau}{2}$

$= \frac{2E\tau}{T} Sa\frac{n\omega\tau}{2}$

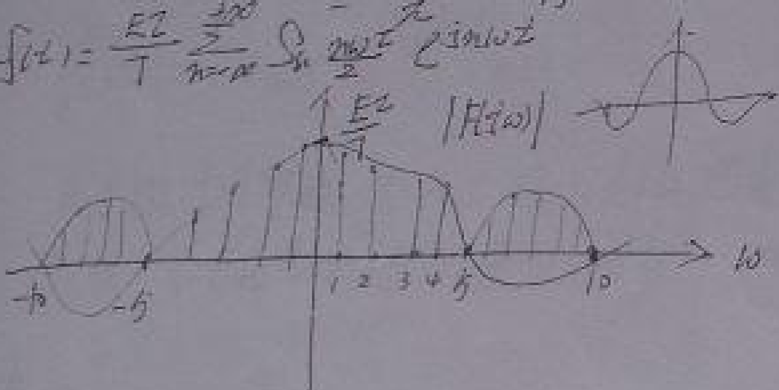
基波振幅：$a_1 = \frac{10}{2\pi}\cdot 2\sin\frac{2\pi}{2}\frac{\tau}{T}$

$= \frac{10}{\pi}\sin\frac{\pi}{5}$

二次谐波：$a_2 = \frac{10}{2\pi}\sin\frac{2}{2}\cdot\frac{2\pi}{5}$

$= \frac{5}{\pi}\sin\frac{2}{5}\pi$

$f(t) = \frac{E\tau}{T}\sum_{n=-\infty}^{\infty} Sa\frac{n\omega\tau}{2}\, e^{jn\omega t}$



四. 1. 由题 特征方程 $\lambda+3=0 \quad \lambda=-3$

∴设 $r_{zi}(t) = Ae^{-3t}$ 则

$A=2$

$r_{zi} = 2e^{-3t}$

2. $H(s) = \frac{1}{s+3}$

∴ $r_{zs}(t) = L^{-1}[\frac{1}{s+2}\cdot\frac{1}{s}]$

$= \frac{1}{2}[u(t) - e^{-2t}u(t)]$

3

$r_{zs}(t) = L^{-1}[\frac{1}{s+3}\cdot\frac{1}{s^2+9}]$

$= L^{-1}[\frac{\frac{1}{18}}{s+3} + \frac{-\frac{1}{18}s+\frac{1}{6}}{s^2+9}]$

$= \frac{1}{18}e^{-3t}u(t) + [(-\frac{1}{18})\cos 3t + \frac{1}{18}\sin 3t]\,u(t)$

∴稳态分量 $r_s(t) = \frac{1}{18}(\sin 3t - \cos 3t)u(t)$

五

1. $E(s) = \frac{E}{s}$

$R_{zs}(s) = E[\frac{1}{6s} - \frac{1}{2(s+2)} + \frac{1}{3(s+3)}]$

$\cancel{H(s) = \frac{\frac{E}{s}}{E[\frac{1}{6s} - \frac{1}{2(s+2)} + \frac{1}{3(s+3)}]}}$

$H(s) = \frac{E[\frac{1}{6s} - \frac{1}{2(s+2)} + \frac{1}{3(s+3)}]}{\frac{E}{s}}$

$= \frac{1}{s^2+5s+6} = \frac{1}{s+2} - \frac{1}{s+3}$

2. 因 H(s) 知方程有特征根为 -2, -3

又由 $r_{zi}(t)$ 知 H(s) 应该有相消的

零极点 -1

∴ $H(s) = \frac{s+1}{(s+2)(s+3)(s+1)} = \frac{s+1}{s^3+6s^2+11s+6}$

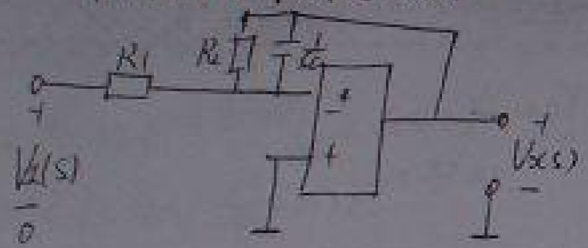
∴系统微分方程为

$r'''(t) + 6r''(t) + 11r'(t) + 6r(t) = e'(t) + e(t)$

3. $H(s) = \frac{1}{s^2+5s+6} = \frac{1}{s+2} - \frac{1}{s+3} = \frac{s^{-1}}{1+2s^{-1}}$

$- \frac{s^{-1}}{1+3s^{-1}}$

七. 取样期间的积分电路为

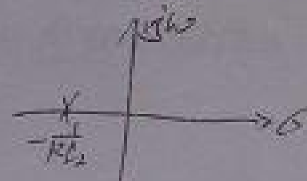


1. $\frac{U_i(s)-0}{R_1} = \frac{0-U_o(s)}{R_2\cdot\frac{1}{sC}/(R_2+\frac{1}{sC})}$

$$\frac{U_o(s)}{U_i(s)} = -\frac{\frac{R_2\frac{1}{sC}}{R_2+\frac{1}{sC}}}{R_1} = -\frac{R_2}{sCR_1R_2+R_1}$$

$$H(s) = -\frac{\frac{1}{CR_1}}{s+\frac{1}{CR_2}}$$

2. 极点 $s=-\frac{1}{CR_2}$



3. $h(t) = -\frac{1}{R_1C}e^{-\frac{1}{R_2C}t}u(t)$

R_2C 为时间常数 R_2C 越小 $h(t)$ 衰减的越快 C 越小 $h(t)$ 初值越大.

八.

1. 由题：

$$R(s) = [E(s)+kR(s)]\cdot\frac{s}{s^2+2s+1}$$

$$R(s)\frac{s^2+2s+1-ks}{s^2+2s+1} = \frac{s}{s^2+2s+1}E(s)$$

$$H(s) = \frac{R(s)}{E(s)} = \frac{s}{s^2+(2-k)s+1}$$

2. 系统要求稳定则极点在左半平面

$2-k>0 \quad k\le 2$

3. 临界稳定时

$$H(s) = \frac{s}{s^2+1}$$

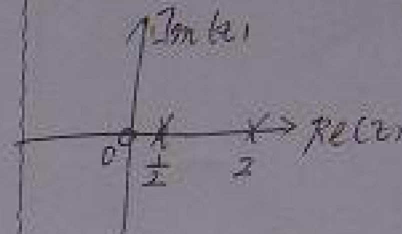
$h(t) = \cos t\, u(t)$

九.

1. $X(z) = \frac{-3z}{2z^2-5z+2} = \frac{-3z}{(z-2)(2z-1)}$

零点 $z=0$

极点 $z_1=2 \quad z_2=\frac{1}{2}$



2. $\frac{X(z)}{z} = \frac{-3}{(z-2)(2z-1)} = \frac{-1}{z-2}+\frac{2}{2z-1}$

$$X(z) = \frac{-z}{z-2}+\frac{z}{z-\frac{1}{2}}$$

① $|z|>2$ 时

$x(n) = [-2^n+(\frac{1}{2})^n]u(n)$

② $|z|<0.5$ 时

$x(n) = -[-(2)^n+(\frac{1}{2})^n]u(-n-1)$

③ $0.5<|z|<2$ 时

$x(n) = (\frac{1}{2})^n u(n) + (2)^n u(-n-1)$

十. 证明：

$$H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}$$

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-jn\omega}$$

$\because h(n)e^{-jn\omega}+h(N-1-n)e^{-j(N-1-n)\omega}$

$= h(n) e^{-j\frac{N-1}{2}\omega} [e^{j(\frac{N-1}{2}\omega - \omega n)} + e^{-j(\frac{N-1}{2}\omega - \omega n)}]$

$= h(n) e^{-j\frac{N-1}{2}\omega} [2\cos(\frac{N-1}{2} - n)]$

$= 2h(n)\cos(\frac{N-1}{2} - n) e^{-j\frac{N-1}{2}\omega}$

∴ 对于 $H(e^{j\omega})$ 来说 $\varphi(\omega) = -\frac{N-1}{2}\omega$，∴该系统具有线性相位

十

解：

~~设系统方程~~

~~$\dot{x}(t) = Ax(t) + Be(t)$~~

~~$\dot{x}(t) = Ax(t) + Be(t)$~~

1. 零输入条件下，设状态转移矩阵 $\Phi(t) = L^{-1}[(SI - A)^{-1}]$

则

$$\begin{bmatrix} 2e^{-2t} - e^{-3t} \\ e^{-2t} - e^{-3t} \end{bmatrix} = \Phi(t) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2e^{-2t} + 2e^{-3t} \\ -e^{-2t} + 2e^{-3t} \end{bmatrix} = \Phi(t) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2e^{-2t} - e^{-3t} & -2e^{-2t} + 2e^{-3t} \\ e^{-2t} - e^{-3t} & -e^{-2t} + 2e^{-3t} \end{bmatrix} = \Phi(t) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Phi(t) = \begin{bmatrix} 2e^{-2t} - e^{-3t} & -2e^{-2t} + 2e^{-3t} \\ e^{-2t} - e^{-3t} & -e^{-2t} + 2e^{-3t} \end{bmatrix}$$

$$L[\Phi(t)] = \begin{bmatrix} \frac{2}{s+2} - \frac{1}{s+3} & -\frac{2}{s+2} + \frac{2}{s+3} \\ \frac{1}{s+2} - \frac{1}{s+3} & \frac{-1}{s+2} + \frac{2}{s+3} \end{bmatrix} = (SI - A)^{-1}$$

2. 由 1 知

$$(SI - A)^{-1} = \frac{1}{(s+2)(s+3)} \begin{bmatrix} 2s+6-s-2 & -2s-6+2s+4 \\ s+3-s-2 & -s-3+2s+4 \end{bmatrix}$$

$$= \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+4 & -2 \\ 1 & s+1 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} s+4 & -2 \\ 1 & s+1 \end{bmatrix}}{(s+2)(s+3)}$$

$$SI - A = \begin{bmatrix} s+1 & 2 \\ -1 & s+4 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} -1 & -2 \\ 1 & -4 \end{bmatrix}$$