2002

一、

1. $\frac{2a}{t^2+a^2}=\frac{1}{aj}\left(\frac{1}{t-aj}-\frac{1}{t+aj}\right)$

∵ $1\overset{F}{\longleftrightarrow}2\pi\delta(\omega)$ $\quad(-jt)\frac{1}{t}\overset{F}{\longleftrightarrow}\frac{dF\{1/t\}}{d\omega}=2\pi(-j)\delta(\omega)$

$\frac{1}{t}\overset{F}{\longleftrightarrow}-j2\pi u(\omega)$

∴ $F\left\{\frac{2a}{t^2+a^2}\right\}=\frac{1}{j}\left[-j2\pi u(\omega)e^{-j(aj)\omega}-(-j2\pi u(\omega))e^{j(aj)\omega}\right]$

$=\left[-2\pi u(\omega)e^{a\omega}+2\pi u(\omega)e^{-a\omega}\right]$

$=2\pi u(\omega)\left[e^{-a\omega}-e^{a\omega}\right]$

2. $F^{-1}\{\delta(\sin\omega)\}$ $\quad\sin\omega=0$ 时 $\omega=k\pi\,(k\in Z)$

则 $\delta(\sin\omega)=\sum_{k=-\infty}^{+\infty}\delta(\omega-k\pi)$

$F[1]=2\pi\delta(\omega)$ $\quad\delta(\omega)\overset{F^{-1}}{\longleftrightarrow}\frac{1}{2\pi}$

∴ $F^{-1}\left\{\sum_{k=-\infty}^{+\infty}\delta(\omega-k\pi)\right\}=\frac{1}{2\pi}\sum_{k=-\infty}^{+\infty}e^{jk\pi t}=$

3. $f(2t-4)u(t-2)=f[2(t-2)]u(t-2)$

$f(2t)\overset{L}{\longleftrightarrow}\frac{1}{|2|}F(\frac{s}{2})=\frac{1}{2}F(\frac{s}{2})$

$f[2(t-2)]u(t-2)\overset{L}{\longleftrightarrow}\frac{1}{2}F(\frac{s}{2})\cdot e^{-2s}$

4. $\frac{s^3+4s^2+5s}{(s+1)^2(s+2)}=\frac{s^3+4s^2+5s+2-2}{s^3+4s^2+5s+2}$

$=1-\frac{2}{(s+1)^2(s+2)}=1-\left[\frac{2}{(s+1)^2}+\frac{-2}{s+1}+\frac{2}{s+2}\right]\overset{L^{-1}}{\longleftrightarrow}\delta(t)-[2te^{-t}-2e^{-t}+2e^{-2t}]u(t)$

$\frac{s^3+4s^2+5s}{(s+1)^2(s+2)}e^{-s}\overset{L^{-1}}{\longleftrightarrow}\delta(t-1)-[2(t-1)e^{-t+1}-2e^{-t+1}+2e^{-2t+4}]u(t-1)$

5. $Z\{nx(n)\}=-z\frac{dX(z)}{dz}$

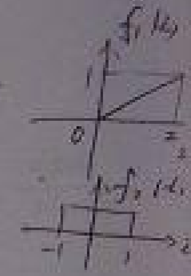
6. $\frac{5.2z^{-1}}{1+4.8z^{-1}-z^{-2}}=\frac{5.2z^{-1}}{(1-0.2z^{-1})(1+5z^{-1})}$

$=\frac{1}{1-0.2z^{-1}}+\frac{-1}{1+5z^{-1}}\longleftrightarrow 0.2^n u(n)+5^n u(-n-1)$

二、

1. $f_1(t)=\frac{1}{2}t[u(t)-u(t-2)]$

$f_2(t)=[u(t+1)-u(t-1)]$



$f(t)=f_1(t)*f_2(t)$

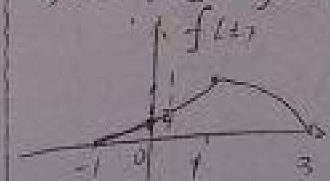
$=\int_{-\infty}^{+\infty}f_1(\tau)f_2(t-\tau)d\tau$

当 $t<0$ 时 $f(t)=\int_0^{1+t}\frac{1}{2}\tau d\tau=\frac{1}{4}(1+t)^2$

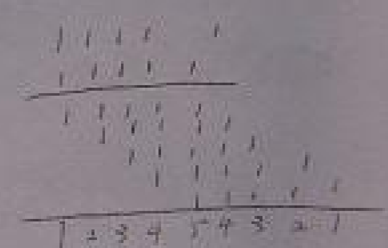
$t<-1$ 时 $f(t)=0$

$1>t>0$ 时 $f(t)=\int_0^{1+t}\frac{1}{2}\tau d\tau=\frac{(1+t)^2}{4}$

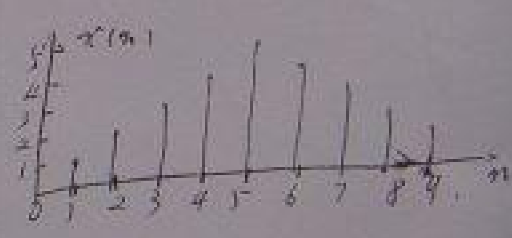
$3>t>1$ 时 $f(t)=\int_{t-1}^{2}\frac{\tau}{2}d\tau=1-\frac{(t-1)^2}{4}$



2. $x_1(n)=u(n)-u(n-5)$ $\quad x_2(n)=u(n-1)-u(n-6)$ 利用乘法确定序列起点为 $n=1$



∴ $x(n)=\delta(n-1)+2\delta(n-2)+3\delta(n-3)+4\delta(n-4)+5\delta(n-5)+4\delta(n-6)+3\delta(n-7)+2\delta(n-8)+\delta(n-9)$



7 由题

$[V_i(s)+V_o(s)]\cdot\frac{s}{s^2+4s+4}\cdot k=V_o(s)$

1 $H(s)=\frac{V_o(s)}{V_i(s)}=\frac{ks}{s^2+(4-k)s+4}$

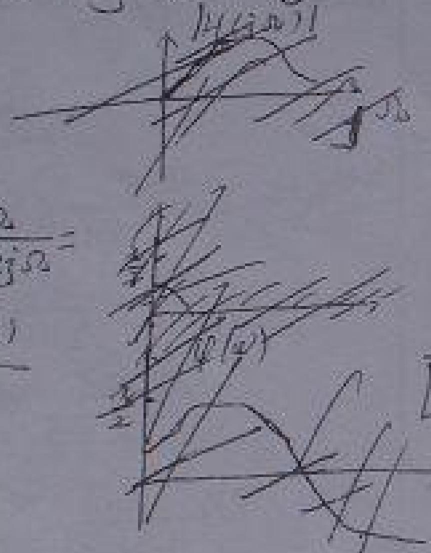
2. 当极点都在S面左边时 $4-k>0$ 时系统稳定 $k\le 4$

3 $k=4$ 时 $H(s)=\frac{4s}{s^2+4}$

$h(t)=4\cos 2t\, u(t)$

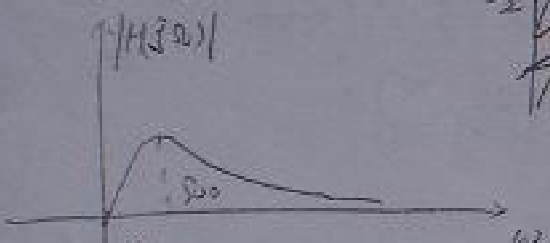
4 $k=0$ 时 $H(s)=\frac{2s}{s^2+2s+4}=\frac{2s}{(s+1+\sqrt{3}j)(s+1-\sqrt{3}j)}$

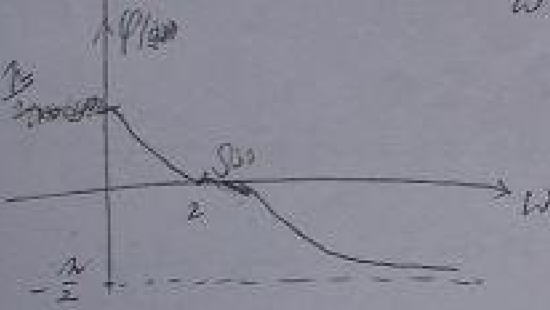
零点 $s=0$ 极点 $s_1=-1-\sqrt{3}j$ $s_2=-1+\sqrt{3}j$

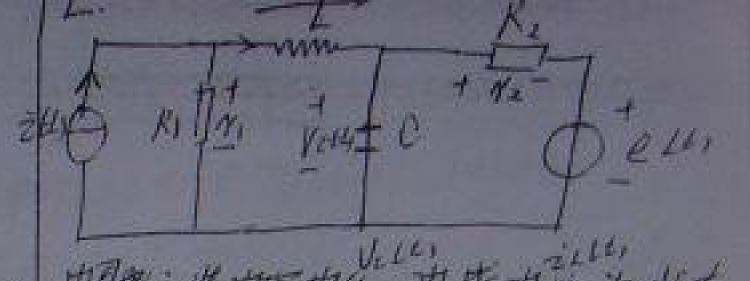


$H(j\Omega)=\frac{2j\Omega}{-\Omega^2+4+2j\Omega}$

$=\frac{4\Omega^2+j(8\Omega-2\Omega^3)}{(4-\Omega^2)^2+4\Omega^2}$







由图：选电容电压，电感电流为状态变量则：列电压方程和电流方程。

$[i(t)-i_L(t)]R_1=L\frac{di_L(t)}{dt}+v_c(t)$

$[i_L(t)-C\frac{dv_c(t)}{dt}]R_2+e(t)=v_c(t)$

则状态方程为

$$\begin{cases} i_L'(t)=-i_L(t)-v_c(t)+i(t)\\ v_c'(t)=i_L(t)-v_c(t)+e(t)\end{cases}$$

输出方程为：

$$\begin{bmatrix}r_1(t)\\r_2(t)\end{bmatrix}=\begin{bmatrix}-1&0\\0&1\end{bmatrix}\begin{bmatrix}i_L(t)\\v_c(t)\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}i(t)\\e(t)\end{bmatrix}$$

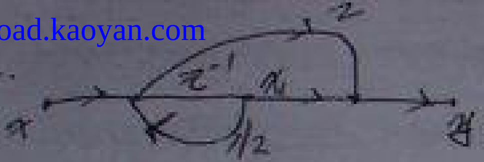
$\Phi(t)=\mathcal{L}^{-1}\{(sI-A)^{-1}\}$

$A=\begin{bmatrix}-1&-1\\1&-1\end{bmatrix}$ $(sI-A)^{-1}=\frac{1}{s^2+2s+2}\begin{bmatrix}s+1&-1\\1&s+1\end{bmatrix}$

$$\Phi(t)=\mathcal{L}^{-1}\left\{\begin{bmatrix}\frac{s+1}{(s+1)^2+1}&\frac{-1}{(s+1)^2+1}\\\frac{1}{(s+1)^2+1}&\frac{s+1}{(s+1)^2+1}\end{bmatrix}\right\}$$

$$=\begin{bmatrix}e^{-t}\cos t&-e^{-t}\sin t\\e^{-t}\sin t&e^{-t}\cos t\end{bmatrix}$$

三.



由题:

~~$x_1(n-1)+\frac{1}{2}x_1(n-1)=x_1(n)$~~

$$\begin{cases}(x+\frac{1}{2}x_1)z^{-1}=x_1 \\ 2x+x_1=y\end{cases}\qquad x_1=\frac{z^{-1}}{1-\frac{1}{2}z^{-1}}x \qquad x\frac{2}{1-\frac{1}{2}z^{-1}}=y$$

$\therefore \frac{Y(z)}{X(z)}=\frac{2}{1-\frac{1}{2}z^{-1}}$ 　1. $y(n)-\frac{1}{2}y(n-1)=2x(n)$

∴ 零输入时 $y(n)=\frac{1}{2}y(n-1)$

$=(\frac{1}{2})^2y(n-2)=(\frac{1}{2})^ny(0)$ 　$y(-1)=2$ 　$y(0)=1$

$\therefore y_{zi}=(\frac{1}{2})^n$

4. $H(z)=\frac{2}{1-\frac{1}{2}z^{-1}}$ 　$X(z)=\frac{1}{1-z^{-1}}$

$Y(z)=\frac{2}{(1-\frac{1}{2}z^{-1})(1-z^{-1})}=\frac{-2}{1-\frac{1}{2}z^{-1}}+\frac{4}{1-z^{-1}}$

$\therefore y_{zs}=-2(\frac{1}{2})^nu(n)+4u(n)$

5. $x(n)=e^{j\frac{\pi}{2}n}=\cos\frac{\pi n}{2}+j\sin\frac{\pi}{2}n=j\sin\frac{\pi n}{2}$

$\sin\frac{\pi n}{2}\xleftrightarrow{Z}\frac{z\sin\frac{\pi}{2}}{z^2-2z\cos\frac{\pi}{2}+1}=\frac{z}{z^2+1}$

$\therefore Y(z)=H(z)\cdot X(z)=\frac{2z}{z-1/2}\cdot\frac{z}{z^2+1}$

$\frac{Y(z)}{z}=\frac{\frac{2}{5}}{z-\frac{1}{2}}+\frac{-\frac{2}{5}z+\frac{4}{5}}{z^2+1}$

~~$Y(z)=\frac{2}{5}\cdot\frac{z}{z-\frac{1}{2}}+\frac{-\frac{2}{5}z^2-\frac{2}{5}+\frac{2}{5}+\frac{4}{5}z}{z^2+1}$~~

~~$y(n)=\frac{2}{5}(\frac{1}{2})^nu(n)-\frac{2}{5}\delta(n)+\left\{\frac{\frac{2}{5}(z^2+1)}{z^2+1}\right\}$~~

$Y(z)=\frac{2}{5}\frac{z}{z-\frac{1}{2}}+\frac{-\frac{2}{5}z^2}{z^2+1}+\frac{\frac{4}{5}z}{z^2+1}$

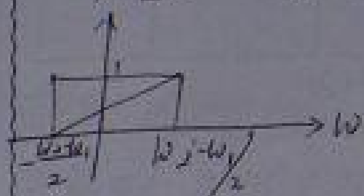
$y(n)=\frac{2}{5}(\frac{1}{2})^nu(n)-\frac{2}{5}\cos\frac{\pi}{2}n+\frac{4}{5}\sin\frac{\pi}{2}n$
$n\geq 0$

$y_s(n)=(-\frac{2}{5}\cos\frac{\pi}{2}n+\frac{4}{5}\sin\frac{\pi}{2}n)u(n)$

四. 由题:

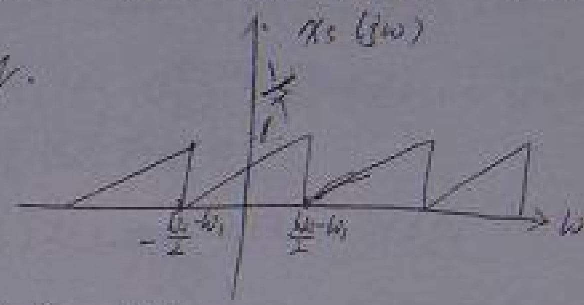
1. $x(t)e^{j\omega_0 t}\xleftrightarrow{F}X[j(\omega-\omega_0)]$

$X[j(\omega-\omega_0)]\cdot H(j\omega)$ 波形如下所示



最大抽样时间间隔 $T=\frac{2\pi}{\omega}=\frac{4\pi}{\omega_2-\omega_1}$

2.



五 解

六.

1. 当 $t>0$ 时

$\frac{1}{C}\int_{-\infty}^{t}i(t)dt+L\frac{di(t)}{dt}+i(t)=10$

$i''(t)+i'(t)+i(t)=0$

2. 由题: $i(0_-)=0$ 　$i'(0_-)=0$

$i(0_+)=i_L(0_+)=0$ 　$i'(0_+)=\frac{1}{C}\int_{-\infty}^{0}i(t)dt$

$=10$

$t>0$ 时 设 $i(t)=A_1e^{\frac{-1+\sqrt{3}j}{2}t}+A_2e^{\frac{-1-\sqrt{3}j}{2}t}$

代入求得:

$i(t)=\frac{20\sqrt{3}}{3}e^{-\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t$ 　$t\geq 0$