2003

一、

1. $f(t) \xleftrightarrow{F} F(\omega)$

$\frac{df(t)}{dt} \xleftrightarrow{F} j\omega F(\omega)$

$-jt\frac{df(t)}{dt} \xleftrightarrow{F} d[j\omega F(\omega)]/d\omega = jF(\omega)+j\omega\frac{dF(\omega)}{d\omega}$

$\therefore t\frac{df(t)}{dt} \xleftrightarrow{F} -F(\omega) - \omega\frac{dF(\omega)}{d\omega}$

2. $sgn(x) = u(x)+u(-x) \xleftrightarrow{F} \frac{2}{j\omega}$

由对偶性

$\frac{2}{jt} \xleftrightarrow{F} 2\pi\, sgn(-\omega)$ sgn(ω)为奇函数

$\therefore F^{-1}\{sgn(\omega)\} = \frac{j}{\pi t}$

3. $W_c Sa(w_c t) = W_c\frac{\sin W_c t}{W_c t} = \frac{\sin W_c t}{t}$

$\sin W_c t \xleftrightarrow{L} \frac{W_c}{s^2+W_c^2}$

$\frac{\sin W_c t}{t} \xleftrightarrow{L} \int_s^{+\infty}\frac{W_c}{s^2+W_c^2}ds$

$= \int_s^{+\infty}\frac{\frac{1}{W_c}}{1+\left(\frac{s}{W_c}\right)^2}ds$

$= \arctan\frac{s}{W_c}\Big|_s^{+\infty}$

$= \frac{\pi}{2} - \arctan\frac{s}{W_c}$

4. $\frac{1}{s^2+9} \xleftrightarrow{L^{-1}} \frac{1}{3}\sin 3t\, u(t)$

$\frac{-2s}{(s^2+9)^2} \xleftrightarrow{L^{-1}} -t\frac{1}{3}\sin 3t\, u(t)$

$\frac{-2s}{(s^2+9)^2}\cdot\frac{-\frac{1}{18}}{s} \longleftrightarrow -\frac{1}{18}\cdot(-\frac{1}{3})\int_{-\infty}^{t} t\sin 3t\,dt\; u(t)$

$= \frac{1}{54}\int_0^t t\sin 3t\,dt$

$= \frac{1}{54}[\frac{1}{9}\sin 3t - \frac{1}{3}t\cos 3t]$

$\therefore \frac{1}{18(s^2+9)^2} \xleftrightarrow{L^{-1}} \frac{1}{108}(\frac{1}{9}\sin 3t - \frac{1}{3}t\cos 3t)$

5. $a^n\sum_{k=0}^{n} b^k = a^n\cdot\frac{1-b^{n+1}}{1-b} = \frac{a^n - b(ab)^n}{1-b}$

$a^n u(n) \xleftrightarrow{Z} \frac{1}{1-az^{-1}}$, $|z|>a$

$(ab)^n u(n) \xleftrightarrow{Z} \frac{1}{1-abz^{-1}}$ $|z|>ab$

$\frac{a^n - b(ab)^n}{1-b}u(n) \xleftrightarrow{Z} \frac{1}{1-b}\left[\frac{1}{1-az^{-1}} - \frac{b}{1-b}\frac{1}{1-abz^{-1}}\right]$

$|z|>|a|$

6. $\frac{1-z^{-1}}{0.5-z^{-1}+0.48z^{-2}} = \frac{2(1-z^{-1})}{1-2z^{-1}+0.96z^{-2}}$

$= \frac{2(1-z^{-1})}{(1-0.8z^{-1})(1-1.2z^{-1})} = \frac{1}{1-0.8z^{-1}} + \frac{1}{1-1.2z^{-1}}$

对$|z|>0.8$

$\therefore Z^{-1}\left\{\frac{1-z^{-1}}{0.5-z^{-1}+0.48z^{-2}}\right\} = 0.8^n u(n) - 1.2^n u(-n)$

二、

$y(t) = h(t) * \delta_T(t)$

$= h(t) * \sum_{k=-\infty}^{+\infty}\delta(t-3k)$

$= \sum_{k=-\infty}^{+\infty} h(t-3k) = \sum_{k=-\infty}^{+\infty} e^{-(t-3k)}u(t-3k)$

$\therefore 3k \le t$ 且 $t\ge 0$ 时

$y(t) = \sum_{k=-\infty}^{0} e^{-(t-3k)}u(t-3k)$

$= \sum_{k=-\infty}^{0} e^{-t+3k} = e^{-t}\frac{1}{1-e^{-3}}$

$= \frac{e^{-t}}{1-e^{-3}}$

三、由题

$f(t) = F^{-1}\{F(\omega)\} = \cos 18\pi t + \cos 12\pi t$

f(t)中各分量最大公约数为6π

∴f(t)是周期信号 周期 $T = \frac{2\pi}{6\pi} = \frac{1}{3}$

$\frac{2\pi}{18\pi} = \frac{1}{9}$ $\frac{2\pi}{12\pi} = \frac{1}{6}$ 周期最大公约数为 $\frac{1}{3}$

f(t)是周期信号 $T = \frac{1}{3}$

解：

1. $R_{xy}(t)=\int_{-\infty}^{+\infty}x(\tau)y^*(\tau-t)d\tau$

$=\int_{-\infty}^{+\infty}x(\tau)y^*[-(t-\tau)]d\tau$

$=x(t)*y^*(-t)$

$y(t)\xleftrightarrow{F}Y(\omega)$

$y^*(t)\xleftrightarrow{F}Y^*(-\omega)$ $y^*(-t)=Y^*(\omega)$

$\therefore$ $R_{xy}(\omega)=X(\omega)\,Y^*(\omega)$

(这里 ω 用 $j\omega$ 表示最合适)

2. $h(t)$ 是因果系统 $\therefore$ $t<0$ 时 $h(t)=0$

$\therefore F\{h(t)u(t)\}=\frac{1}{2\pi}\{F\{h(t)\}*F\{u(t)\}\}$

$=\frac{1}{2\pi}\{[R(\omega)+jI(\omega)]*[\pi\delta(\omega)+\frac{1}{j\omega}]\}$

$=\frac{1}{2\pi}\{\pi R(\omega)+j\pi I(\omega)+\frac{1}{j}R(\omega)*\frac{1}{\omega}+$

$I(\omega)*\frac{1}{\omega}\}=R(\omega)+jI(\omega)$

$\therefore\begin{cases}\frac{\pi R(\omega)}{2\pi}+\frac{1}{2\pi}I(\omega)*\frac{1}{\omega}=R(\omega)\\ \frac{\pi I(\omega)}{2\pi}+-R(\omega)*\frac{1}{\omega}=I(\omega)\end{cases}$

可得 $R(\omega)=\frac{1}{\pi}\int_{-\infty}^{+\infty}I(\lambda)\frac{1}{\omega-\lambda}d\lambda$

$I(\omega)=-\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{R(\lambda)}{\omega-\lambda}d\lambda$

五 ① 由题：

$\frac{V_1(s)-0}{1}=\frac{0-V_0(s)}{\frac{1}{s}R_2/\frac{1}{s}+R_2}$

$V_0(s)=-V_1(s)\cdot(\frac{1}{s}R_2)/(\frac{1}{s}+R_2)$

$=-(\frac{1}{s}-\frac{e^{-s}}{s})(\frac{R_2}{1+sR_2})$

~~$-\frac{1}{s^2}+\frac{e^{-s}}{s}-\frac{R_2}{s}\frac{e^{-s}}{s}$~~

~~$V_0(t)=-t u(t)+(t-1)u(t-1)-R_2 u(t)+R_2 u(t-1)$~~

~~$=-(t+R_2)u(t)+(t-1+R_2)u(t-1)$~~

~~② $R_2\to\infty$ 时相当于~~

$V_0(s)=-(1-e^{-s})(\frac{R_2}{s}-\frac{R_2}{s+\frac{1}{R_2}})$

$=-\frac{1-e^{-s}}{s}R_2+\frac{1-e^{-s}}{s+\frac{1}{R_2}}R_2$

$V_0(t)=-R_2[u(t)-u(t-1)]+R_2e^{-\frac{1}{R_2}t}u(t)$

$-R_2e^{-\frac{1}{R_2}(t-1)}u(t-1)$

② (分析一)

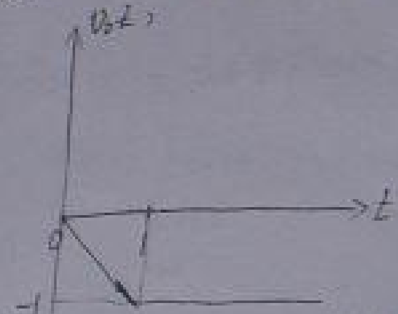
$V_0(t)=R_2(e^{-\frac{1}{R_2}t}-1)u(t)+R_2(1-e^{-\frac{1}{R_2}(t-1)})u(t-1)$

$=R_2[1-(\frac{t}{R_2})+(\frac{-t}{R_2})^2\cdot\frac{1}{2}+\cdots-1]u(t)$

$+R_2[1-\frac{t-1}{R_2}+(\frac{t-1}{R_2})^2\cdot\frac{1}{2}+\cdots]u(t-1)$

当 $R\to+\infty$ 时

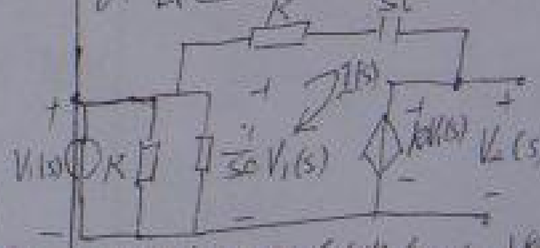
$V_0(t)=-t u(t)+(t-1)u(t-1)$



(分析二) $R_2\to\infty$ 将其看成开路则

$V_0(s)=\frac{-V_1(s)}{\frac{1}{s}}$ 得上述同样结果

2. 由题：



图中的反馈系统模型为：

七

1. $y(n)=\sum_{k=0}^{7}x(n-k)$

2. $H(z)=\sum_{k=0}^{7}z^{-k}X(z)\Big/X(z)=\sum_{k=0}^{7}z^{-k}$

$=\frac{1-z^{-8}}{1-z^{-1}}$

3. $H(z)=\frac{(1+z^{-4})(1+z^{-2})(1+z^{-1})(1-z^{-1})}{1-z^{-1}}=1+z^{-1}+z^{-2}$

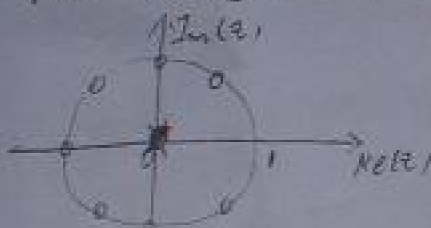
$+z^{-3}+z^{-4}+z^{-5}+z^{-6}+z^{-7}$

$\therefore h(n)=\sum_{k=0}^{7}\delta(n-k)$

4. $H(z)=\frac{1-z^{-8}}{1-z^{-1}}=\frac{z^{8}-1}{z^{8}-z^{7}}=\frac{(z^{4}+1)(z^{2}+1)(z+1)}{z^{7}}$

零点 $z=0$ (7阶)

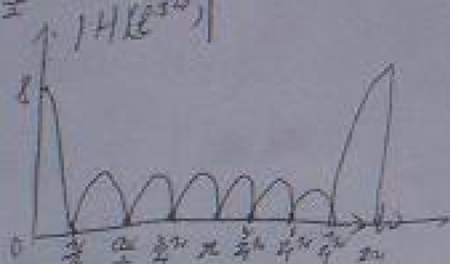
极点 -1, $\pm j$, $e^{\pm j\frac{\pi}{4}}$, $e^{\pm j\frac{3}{4}\pi}$



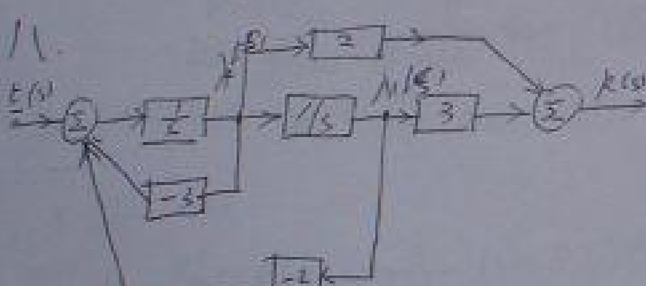
$H(e^{j\omega})=H(z)\big|_{z=e^{j\omega}}=\frac{1-e^{-j8\omega}}{1-e^{-j\omega}}$

$=\frac{e^{j4\omega}-e^{-j4\omega}}{e^{j\frac{1}{2}\omega}-e^{-j\frac{1}{2}\omega}}\cdot\frac{e^{-j4\omega}}{e^{-j\frac{1}{2}\omega}}$

$=\frac{\sin 4\omega}{\sin\frac{\omega}{2}}\cdot e^{-j\frac{7}{2}\omega}$



八.



列方程:

$\lambda_1'(t)=\lambda_2(t)$

$\lambda_2'(t)=-2\lambda_1(t)-3\lambda_2(t)+e(t)$

$\begin{bmatrix}\lambda_1'(t)\\ \cdots\end{bmatrix}=\begin{bmatrix}0&1\\ \cdots&\cdots\end{bmatrix}\begin{bmatrix}\lambda_1(t)\\ \cdots\end{bmatrix}+\begin{bmatrix}0\\ \cdots\end{bmatrix}e(t)$

$A=\begin{bmatrix}0&1\\-2&-3\end{bmatrix}$

$e^{At}=\mathcal{L}^{-1}\{(sI-A)^{-1}\}$

$=\mathcal{L}^{-1}\left\{\begin{bmatrix}s&-1\\2&s+3\end{bmatrix}^{-1}\right\}$

$=\mathcal{L}^{-1}\left\{\frac{1}{s^2+3s+2}\begin{bmatrix}s+3&1\\-2&s\end{bmatrix}\right\}$

$=\mathcal{L}^{-1}\left\{\begin{bmatrix}\frac{2}{s+1}+\frac{-1}{s+2}&\frac{1}{s+1}+\frac{-1}{s+2}\\ \frac{-2}{s+1}+\frac{2}{s+2}&\frac{-1}{s+1}+\frac{2}{s+2}\end{bmatrix}\right\}$

$=\begin{bmatrix}2e^{-t}-e^{-2t}&e^{-t}-e^{-2t}\\ -2e^{-t}+2e^{-2t}&-e^{-t}+2e^{-2t}\end{bmatrix}\cdot\varepsilon(t)$

四题中模型:

$V_2(s)=[V_1(s)-V_2(s)] \quad K\cdot\frac{R\cdot\frac{1}{sC}/(R+\frac{1}{sC})}{R+\frac{1}{sC}+R\cdot\frac{1}{sC}/(R+\frac{1}{sC})}$

$H(s)=\frac{V_2(s)}{V_1(s)}=\frac{KsCR}{(sCR+1)^2+sCR+KsCR}$

$=\frac{KsCR}{s^2C^2R^2+(3+K)sCR+1}$

若能产生振荡则 ~~sCR~~ $H(s)$ 极点实部为0

故 $k=-3$

此时 $H(s)=\frac{KsCR}{s^2+\frac{1}{R^2C^2}}$

$\omega_0=\sqrt{\frac{1}{RC}}$

5. 内题:

① $R(s)=2[\frac{R(s)}{2}\cdot(-3)+E(s)]\cdot\frac{1}{s}$

∴ $H(s)=\frac{R(s)}{E(s)}=\frac{2}{s+3}$

$h(t)=2e^{-3t}u(t)$

② $R(s)=E(s)H(s)$

$=\frac{3}{s^2+9}\cdot\frac{2}{s+3}$

$=\frac{-\frac{1}{3}s+1}{s^2+9}+\frac{\frac{1}{3}}{s+3}=-\frac{1}{3}\frac{s}{s^2+9}+\frac{1}{3}\frac{3}{s^2+9}+\frac{1}{3}\frac{1}{s+3}$

∴ $r_s(t)=(-\frac{1}{3}\cos 3t+\frac{1}{3}\sin 3t)u(t)$

六、解题:

1. $u(n)\overset{Z}{\longleftrightarrow}\frac{1}{1-z^{-1}} \quad |z|>1$

$nu(n)\overset{Z}{\longleftrightarrow}-z\frac{d}{dz}(\frac{1}{1-z^{-1}})$

$=\frac{z}{(z-1)^2}=\frac{z^{-1}}{1-2z^{-1}+z^{-2}} \quad |z|>1$

$(\frac{1}{2})^n u(n)\overset{Z}{\longleftrightarrow}\frac{1}{1-(\frac{1}{2})z^{-1}} \quad |z|>\frac{1}{2}$

∴ $H(z)=\frac{Y(z)}{X(z)}$

$=\frac{3(\frac{1}{1-z^{-1}}-\frac{1}{1-\frac{1}{2}z^{-1}})-\frac{z^{-1}}{1-2z^{-1}+z^{-2}}}{\quad}$

$=3\frac{\frac{1}{2}z^{-1}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})}\cdot\frac{(1-z^{-1})^2}{z^{-1}}-1$

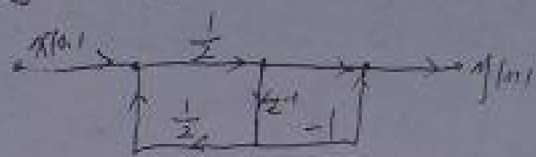
$=\frac{\frac{3}{2}(1-z^{-1})-1+\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}=\frac{\frac{1}{2}-z^{-1}}{1-\frac{1}{2}z^{-1}}$

$H(z)=\frac{-\frac{3}{2}+2-z^{-1}}{1-\frac{1}{2}z^{-1}} \quad |z|>1$

$h(n)=-\frac{3}{2}(\frac{1}{2})^n u(n)+2\delta(n)$

2. 差分方程为:

$y(n)-\frac{1}{2}y(n-1)=\frac{1}{2}x(n)-x(n-1)$



3. $y(n)-\frac{1}{2}y(n-1)=\frac{1}{2}x(n)-x(n-1)$

两边同时取z变换:

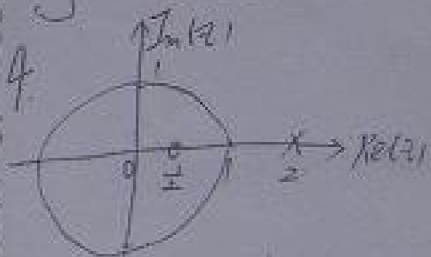
$Y(z)-\frac{1}{2}z^{-1}[Y(z)+y(-1)z]=\frac{1}{2}X(z)-X(z)z^{-1}$

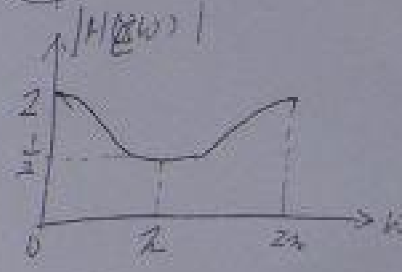
$Y_{zi}(z)=\frac{\frac{1}{2}y(-1)}{1-\frac{1}{2}z^{-1}}=\frac{3}{1-\frac{1}{2}z^{-1}}$

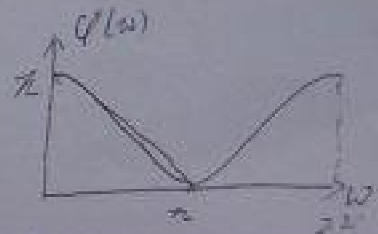
$y_{zi}(n)=3(\frac{1}{2})^n u(n)$

∴ $y(n)=3u(n)-nu(n)$

4.







带通滤波器