2004

一、

判断是否为周期信号，根据其展开式中各分量角频率是否存在最大公约数 因子：（或者是周期存在最小公倍数）求数

$\omega_1 = 25\pi$  $\omega_2 = 15\pi$  $\omega_3 = 1\pi$

$\therefore f_1(t) = x_1(t) + x_2(t)$ 不是周期信号

$f_2(t) = x_1(t) + x_3(t)$ 是周期信号且 $T=$

$\frac{2\pi}{15\pi}\times 5 = \frac{2}{3}\pi$

$f_3(t) = x_2(t) + x_4(t)$ 不是周期信号

二、

$F(\omega) = \frac{8}{\omega}\sin(5\times10^{-6}\omega) = \frac{8\times5\times10^{-6}}{5\times10^{-6}\omega}\sin 5\times10^{-6}\omega$

$= 40\times10^{-6}\, Sa(5\times10^{-6}\omega) = E\tau\, Sa\frac{\omega\tau}{2}$

$\tau = 10\times10^{-6}$  $E=4$  （分析一）：

$f_T(t) = f(t) * \sum_{k=-\infty}^{+\infty}\delta(t-kT)$

$F_T(\omega) = F(\omega)\cdot\omega_0\sum_{k=-\infty}^{+\infty}\delta(\omega-k\omega_0)$

$=\omega_0\sum_{k=-\infty}^{+\infty}F(k\omega_0)$

$=\omega_0\sum_{k=-\infty}^{+\infty}\frac{8}{k\omega_0}\sin 5\times10^{-6}k\omega_0$

$k=0$ 时为平均分量 $E_0' = 8\omega_0\cdot5\times10^{-6} = 8\cdot\frac{2\pi}{40\times10^{-6}}\times5\times10^{-6} = 2\pi$

$k=2$ 时 $E_2' = 8\omega_0\cdot\frac{8}{2\omega_0}\sin 5\times10^{-6}\times2\times\frac{2\pi}{40\times10^{-6}}$

$= 4$

$k=4$ 时 $E_4' = \omega_0\frac{8}{4\omega_0}\sin 5\times10^{-6}\times4\,\frac{2\pi}{40\times10^{-6}}$

$=0$

由周期信号的谐和傅里叶级数的关系知：

（分析二）由傅氏级数知：

$f(t) = 4[u(t+\frac{\tau}{2}) - u(t-\frac{\tau}{2})]$

$\therefore E_0 = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} f(t)\,dt$

$= \frac{4}{40\times10^{-6}}\times10^{-5} = 1$

$E_2 = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} f(t)\cos 2\omega_0 t\,dt$

$= \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} E\cos 2\omega_0 t\,dt$

$= \frac{E}{T}\frac{1}{2\omega_0}\sin 2\omega_0 t\Big|_{-\frac{\tau}{2}}^{\frac{\tau}{2}}$

$= \frac{E\sin\omega_0\tau}{\omega_0 T} = \frac{4}{2\pi} = \frac{2}{\pi}$

$E_4 = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} f(t)\cos 4\omega_0 t\,dt$

$=0$

三、$f(t)\xleftrightarrow{F}F(\omega)$

1. $f(2t)\xleftrightarrow{F}\frac{1}{|2|}F(\frac{\omega}{2}) = \frac{1}{2}\frac{1}{\frac{\omega^2}{4}+1}$

2. $f(t-2)\xleftrightarrow{F}F(\omega)e^{-j2\omega} = \frac{e^{-j2\omega}}{\omega^2+1}$

3. $f(-t)\xleftrightarrow{F}\frac{1}{|-1|}\cdot F(-\omega) = F_1(\omega) = \frac{1}{\omega^2+4}$

$f(-t)e^{jt}\xleftrightarrow{F}F_1(\omega-1) = \frac{1}{(\omega-1)^2+4}$

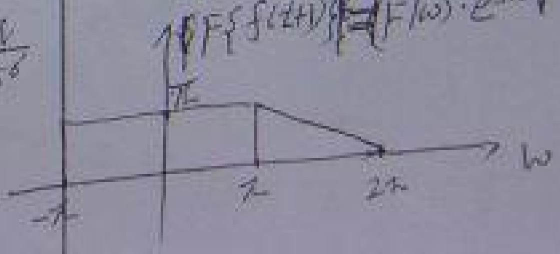
4. $\frac{df(t)}{dt}\xleftrightarrow{F}(j\omega)F(\omega) = \frac{j\omega}{\omega^2+1}$

四、$f(t+1) = 2f_1(t+1)f_2(t+1)$

$F\{f(t+1)\} = \frac{1}{\pi}F\{f_1(t+1)\} * F\{f_2(t+1)\}$

$=\frac{1}{\pi}[u(\omega+\frac{\pi}{2}) - u(\omega-\frac{\pi}{2})] * [u(\omega+\frac{3\pi}{2}) - u(\omega-\frac{3\pi}{2})]$ 其幅频响应如下图所示.

$|F\{f(t+1)\}| = |F(\omega)\cdot e^{j\omega}|$



$\int_{-\infty}^{\infty} f(t) e^{j\pi t} dt = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \Big|_{\omega=-\pi}$

$= F(-\pi) = -\pi$.

$y(t) = f(t) * \frac{\sin t}{t} = f(t) * Sa\, t$

$Sa(t) = A \frac{w_c}{\pi} Sa\, w_c t \quad A = \pi \quad w_c = 1$

$Y(\omega) = \frac{1}{2\pi} F(\omega) * \pi [u(\omega+1) - u(\omega-1)]$

$= \frac{1}{2} F(\omega) * [u(\omega+1) - u(\omega-1)]$

$= \frac{\pi}{2} [u(\omega+1) - u(\omega-1)] e^{-j\omega}$

故 $y(t) = \frac{\pi}{2} \cdot \frac{1}{\pi} Sa(t-1) = \frac{1}{2} Sa(t-1)$

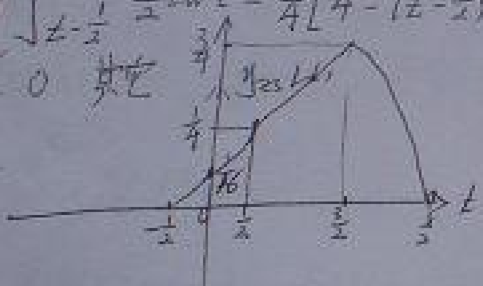
五.

1. $y_{zs}(t) = h(t) * e(t)$

$= \int_{-\infty}^{+\infty} \frac{1}{2}\tau [u(\tau) - u(\tau-2)][u(t-\tau+\frac{1}{2}) - u(t-\tau-\frac{1}{2})]$

$= \int_{0}^{2} \frac{1}{2}\tau [u(t-\tau+\frac{1}{2}) - u(t-\tau-\frac{1}{2})] d\tau$

$= \begin{cases} \int_0^{t+\frac{1}{2}} \frac{1}{2}\tau d\tau = \frac{1}{4}(t+\frac{1}{2})^2 & -\frac{1}{2} \le t \le \frac{1}{2} \\ \int_{t-\frac{1}{2}}^{t+\frac{1}{2}} \frac{1}{2}\tau d\tau = \frac{1}{4}[(t+\frac{1}{2})^2 - (t-\frac{1}{2})^2] = \frac{1}{4} \cdot 2t = \frac{t}{2} & (\frac{3}{2} \ge t \ge \frac{1}{2}) \\ \int_{t-\frac{1}{2}}^{2} \frac{1}{2}\tau d\tau = \frac{1}{4}[4 - (t-\frac{1}{2})^2] = 1 - \frac{1}{4}(t-\frac{1}{2})^2 & (\frac{5}{2} \ge t \ge \frac{3}{2}) \\ 0 & 其它 \end{cases}$

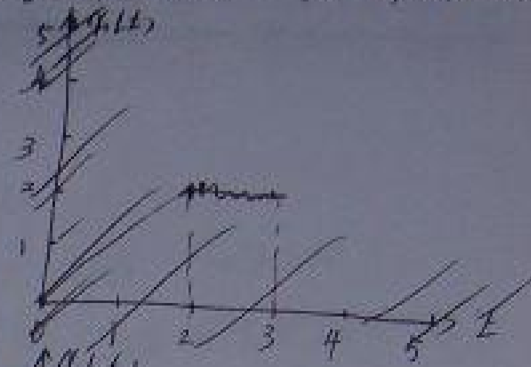


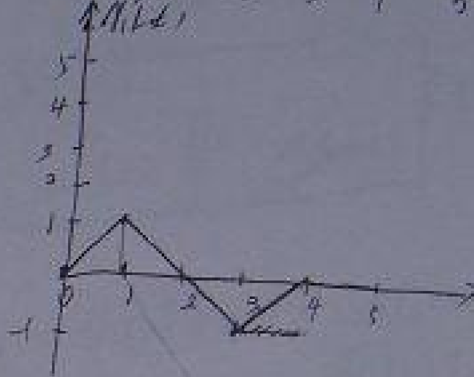

2. $H(s) = \dfrac{2\,\frac{1-2e^{-s}+e^{-2s}}{s^2}}{\frac{1-e^{-2s}}{s}} = \dfrac{1-e^{-s}}{s(1+e^{-s})}$

$R_1(s) = E_1(s) H(s) = (\frac{1-2e^{-s}+e^{-2s}}{s}) \cdot \frac{1-e^{-s}}{s(1+e^{-s})}$

$= \dfrac{(1+e^{-s})(1-e^{-s})^3}{(1-e^{-2s})(1+e^{-2s}-2e^{-s})}$

$y_1(t) = 2u(t) - 2(t-1)u(t-1)$
$+2(t-3)u(t-3) - (t-4)u(t-4)$



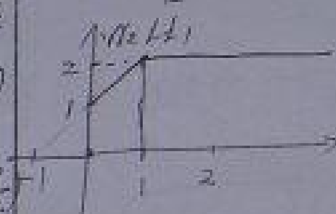


$R_2(s) = E_2(s) H(s) = (\frac{e^{s}+1-e^{-s}-e^{-2s}}{s})$
$\cdot \frac{1-e^{-s}}{s(1+e^{-s})} = \frac{1}{s^2}[\frac{e^{s}(1-e^{-2s})+(1-e^{-2s})}{1+e^{-s}}]$

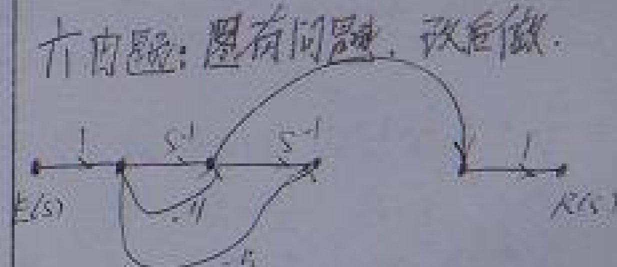
$= \frac{1}{s^2}[(e^{s}+1)(1-e^{-s})]$

$\frac{1}{s^2}(e^{s}-1+1-e^{-s})$

$y_2(t) = [\ \ + (t+1)u(t+1) - (t-1)u(t-1)$



六两题：图有问题，改后做.



1. $y'(t) = -4y(t) + e(t)$

2. 由 $H(s) = \frac{R(s)}{E(s)} = \frac{4s+1}{s}$

$h_2(t) = -4\delta(t) + u(t)$

3. 特征方程：2　题参照06年流图

1. 由流图

$$\begin{cases} x_1' = y(t) \\ x'(t) = e(t) - 4x(t) - 5x_1 \\ \therefore x'' = e'(t) - 4x'(t) - 5x(t) \end{cases}$$

系统方程 $x'' + 4x'(t) + 5x(t) = e'(t)$

2. $H(s) = \frac{R(s)}{E(s)} = \frac{s}{s^2+4s+5}$

3. 系统特征方程：$\alpha^2 + 4\alpha + 5 = 0$

$\alpha = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm j$

$r_{zi}(t) = A_1 e^{(-2-j)t} + B_1 e^{(-2+j)t}$

代入 $r(0) = r'(0) = 1$

$$\begin{cases} A_1 + B_1 = 1 \\ (-2-j)A_1 + (-2+j)B_1 = 1 \end{cases}$$ 得 $A_1 = \frac{1+3j}{2}$，$B_1 = \frac{1-3j}{2}$

$\therefore r_{zi}(t) = e^{-2t}\left[\frac{1+3j}{2} e^{-jt} + \frac{1-3j}{2} e^{jt}\right]$

$= e^{-2t}\left[\cos t + \frac{3j}{2}(-2j\sin t)\right]$

$= e^{-2t}(\cos t + 3\sin t)$

4. $e(t) = u(t)$　$E(s) = \frac{1}{s}$

$\therefore R(s) = H(s)E(s) = \frac{s}{s^2+4s+5} \cdot \frac{1}{s} = \frac{1}{(s+2)^2+1}$

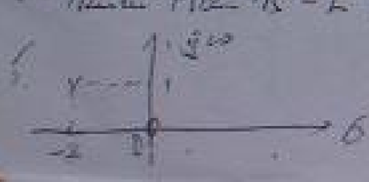
$r_{zs}(t) = e^{-2t}\sin t \, u(t)$

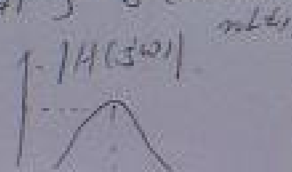
5. $e(t) = \sin t$　$E(s) = \frac{1}{s^2+1}$

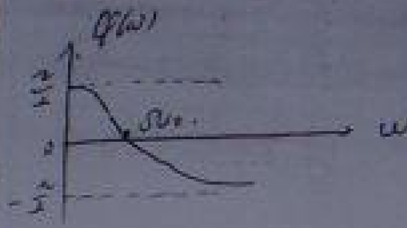
$\therefore R(s) = \frac{s}{(s^2+1)(s^2+4s+5)} = \frac{\frac{1}{8}(s+1)}{s^2+1} + \frac{-\frac{1}{8}s + \frac{-5}{8}}{s^2+4s+5}$

$\therefore$ 稳态响应 $r_s = \mathcal{L}^{-1}\left\{\frac{\frac{1}{8}(s+1)}{s^2+1}\right\} = \frac{1}{8}(\cos t + \sin t)\, u(t)$

6.



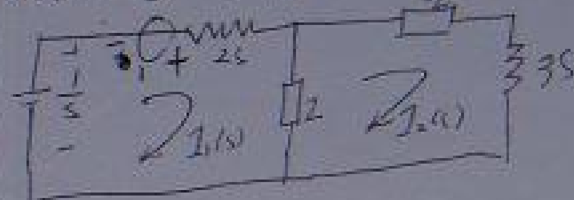




七. 由题：

$i_1(0_-) = \frac{1}{2} = 0.5A$　$i_2(0_-) = 0$

$i_L(0_+) = \frac{1}{3}$　S 域 相关电路如下：



得：

$\frac{1}{s} + 1 = I_1(s)(2s+2) - 2I_2(s)$

$0 = -I_1(s) \times 2 + (4+3s)I_2(s)$

~~……~~ $I_2(s) = \frac{1}{s(s+2)(3s+1)}$

$= \frac{\frac{1}{2}}{s} + \frac{\frac{1}{10}}{s+2} + \frac{-\frac{9}{5}}{3s+1}$

~~……~~

$i_2(t) = \left(\frac{1}{2} + \frac{1}{10} e^{-2t} - \frac{3}{5} e^{-\frac{1}{3}t}\right) u(t)$

八.

解：1. $H(z) = \frac{1 + \frac{1}{2}z^{-1}}{1 + \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}$

$H(z) = \frac{1 + \frac{1}{2}z^{-1}}{(1+\frac{1}{3}z^{-1})(1+\frac{1}{2}z^{-1})} = \frac{1}{1+\frac{1}{3}z^{-1}}$

$h(n) = (-\frac{1}{3})^n u(n)$

2. 特征方程 $\alpha^2 + \frac{5}{6}\alpha + \frac{1}{6} = 0$

求 $\alpha_1 = -\frac{1}{2}$　$\alpha_2 = -\frac{1}{3}$

$y_{zi}(n) = C_1(-\frac{1}{2})^n + C_2(-\frac{1}{3})^n$

$$\begin{cases} C_1 + C_2 = -1 \\ -\frac{1}{2}C_1 + (-\frac{1}{3})C_2 = -\frac{1}{6} \end{cases}$$ $C_1 = -3$，$C_2 = 4$

$y_{zi}(n) = [(-3)(-\frac{1}{2})^n + 4(-\frac{1}{3})^n] u(n)$

九.

$$H(z)=\frac{-3z^{-1}}{1-\frac{5}{2}z^{-1}+z^{-2}}=\frac{-2}{1-2z^{-1}}+\frac{2}{1-\frac{1}{2}z^{-1}}$$

1 系统是因果的

此时 $h(n)=-2(2)^n u(n)+2(\frac{1}{2})^n u(n)$

$|z|>2$

2 系统是非因果的分两种情况

① $h(n)=[(2)2^n+2(\frac{1}{2})^n]u(-n-1)$

$=[2^{n+1}+-(\frac{1}{2})^{n-1}]u(-n-1)$

$|z|<\frac{1}{2}$

② $h(n)=2(\frac{1}{2})^n u(n)+2\cdot 2^n u(-n-1)$

$=\frac{1}{2^{n-1}}u(n)+2^{n+1}u(-n-1)$

3 系统是稳定的,其收敛域包括单位圆

$h(n)=\frac{1}{2^{n-1}}u(n)+2^n u(-n-1)$

十 由题意:

$I_s(t)=\lambda_2'(t)+\lambda_1(t)$

$\lambda_2(t)+\lambda_2'(t)=\lambda_1'(t)+V_s(t)$

整理:

$$\begin{cases}\lambda_1'(t)=-\lambda_1(t)+\lambda_2(t)+I_s(t)-V_s(t)\\ \lambda_2'(t)=-\lambda_1(t)+I_s(t)\end{cases}$$

$$A=\begin{bmatrix}-1 & 1\\ -1 & 0\end{bmatrix}$$

$$\therefore \Phi(t)=L^{-1}\{(sI-A)^{-1}\}$$

$$=L^{-1}\left\{\begin{bmatrix}s+1 & -1\\ 1 & s\end{bmatrix}^{-1}\right\}$$

$$=L^{-1}\left\{\frac{1}{s^2+s+1}\begin{bmatrix}s & 1\\ -1 & s+1\end{bmatrix}\right\}$$

$$=\begin{bmatrix}e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t+\frac{1}{\sqrt{3}}e^{-\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t & \frac{2}{\sqrt{3}}e^{-\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t\\ -\frac{2}{\sqrt{3}}e^{-\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t & e^{-\frac{1}{2}t}(\cos\frac{\sqrt{3}}{2}t+\frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t)\end{bmatrix}u(t)$$