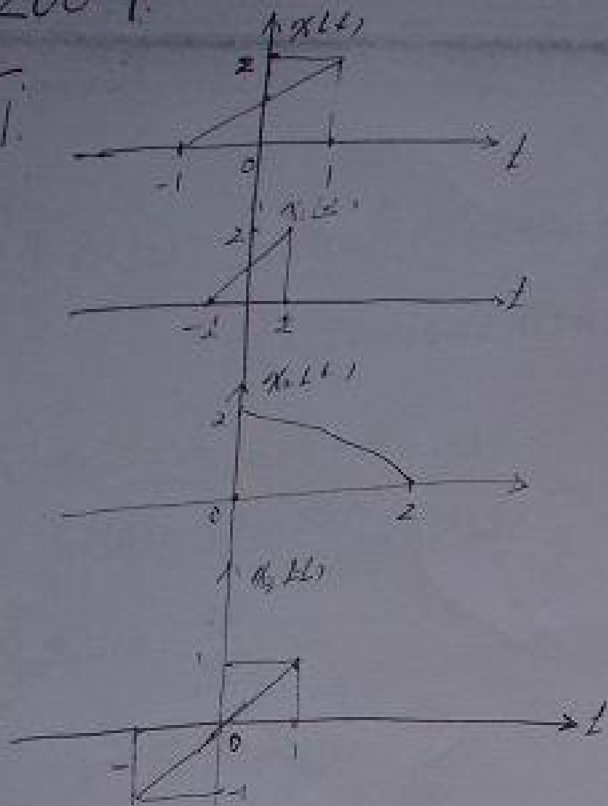
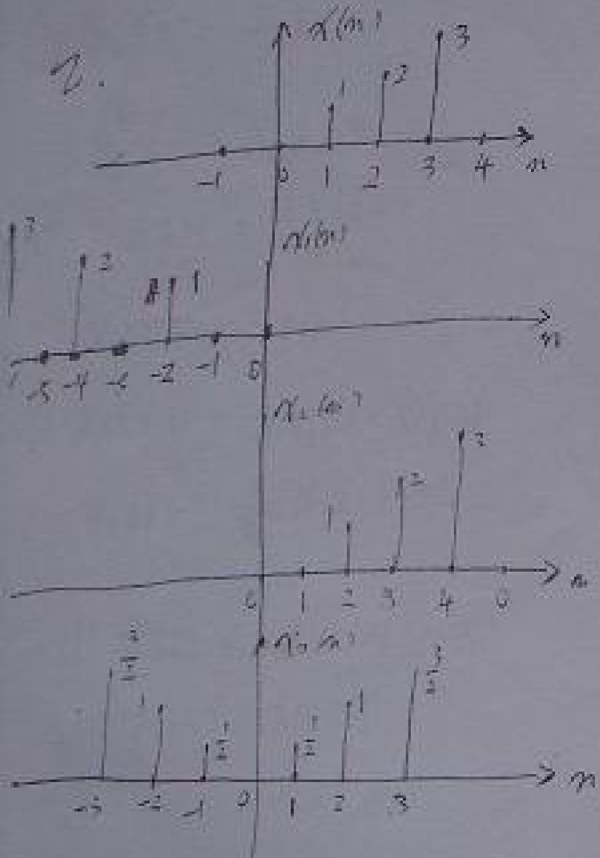
2007

一、

1.



2.



3. ① $\int_{-\infty}^{\infty} Sa(t)\delta(t-\pi)\,dt$

$= \int_{-\infty}^{\infty} \frac{\sin t}{t}\delta(t-\pi)\,dt$

$= 0$

② $\int_0^t u(t-\tau)\,d\tau$

$= t u(t)$

③ $\int_{-\infty}^{+\infty} Sa(\tau)\delta(t-\tau)\,d\tau$

$= Sa(t) * \delta(t) = Sa(t)$

④ $\int_{-\infty}^{t} \sin\Omega_0\tau\, u(t-\tau)\,d\tau$

$= \int_{-\infty}^{+\infty} \sin\Omega_0\tau\, u(t-\tau)\,d\tau$

$= \sin\Omega_0 t * u(t)$ 积分.

$= \frac{\cos\Omega_0 t}{\Omega_0}$

或者 $\sin\Omega_0 t \leftrightarrow j\pi[\delta(\Omega+\Omega_0)-\delta(\Omega-\Omega_0)]$

$u(t) \leftrightarrow \pi\delta(\Omega) + \frac{1}{j\Omega}$

$\therefore \sin\Omega_0 t * u(t) \leftrightarrow -\pi\frac{1}{\Omega_0}[\delta(\Omega-\Omega_0)+\delta(\Omega$

$-\frac{\cos\Omega_0 t}{\Omega_0}$

4.

① $y(t) = ax(t) + b$

t时刻输出只和t时刻输入有关

∴因果 a, b有限∴稳定 有常数项b的

∴非线性 $y(t-t_0) = ax(t-t_0)+b$ ∴时不变

② $y(t) = x(2t)$

非因果, 稳定, 线性, 时变

③ $y(n) = n x(n-1)$

因果, 稳定, 线性, 时不变

④ $y(n) = 2^n x(n)$

非因果, 不稳定, 线性, 时变.

5. ① $2x(t) \longleftrightarrow 2y(t)$

② $\int_{-\infty}^{t} y(\tau - t_0)\,d\tau$

③ $y(n) - 2y(n-1)$

$\sum_{m=-\infty}^{n} y(m)$

二

1 (1) $X_1(j\Omega) = \frac{1}{2\pi} X(j\Omega) * [\pi\delta(\Omega+\Omega_0) + \pi\delta(\Omega-\Omega_0)]$

$= \frac{1}{2}[X(j\Omega+j\Omega_0) + X(j\Omega-j\Omega_0)]$

(2) $x_2(t) = x(t+1) + x(t-1)$

$X_2(j\Omega) = X(j\Omega)e^{j\Omega} + X(j\Omega)e^{-j\Omega}$

(3) $x_3(t) = \frac{dx(t)}{dt}$

$X_3(j\Omega) = j\Omega X(j\Omega)$

2

(1) $X_1(j\Omega) = \frac{1}{2} X(j\frac{\Omega}{2})$

$x_1(t) = x(\frac{1}{2}t)$

(2) $X_2(j\Omega) = j\frac{dX(j\Omega)}{d\Omega}$

$x_2(t) = j(-jt)x(t) = t x(t)$

(3) $X_3(j\Omega) = j X(j\Omega) \cdot X(j\Omega)$

$x_3(t) = x(t) * x(t)$

3

(1) $x_1(t) = x(t-t_0)$

$X_1(s) = X(s)e^{-st_0}$

(2) $x_2(t) = x(t)e^{s_0 t}$

$X_2(s) = X(s-s_0)$

(3) $x_3(t) = x'(t)$

$X_3(s) = sX(s) - x(0)$

4

(1) $X_1(s) = X(s-s_0)$

$x_1(t) = x(t)e^{s_0 t}$

(2) $X_2(s) = -\frac{dX(s)}{ds}$

$x_2(t) = -(-t)x(t) = t x(t)$

(3) $X_3(s) = X(\frac{s}{2}) = 2 \cdot \frac{1}{2}X(\frac{s}{2})$

$x_3(t) = 2x(2t)$

5. (1) $x_1(n) = \left(\frac{1}{2}\right)^n u(n-1)$

$X_1(z) = \sum_{n=-\infty}^{+\infty} x_1(n) z^{-n}$

$= \frac{1}{2}z^{-1} + \left(\frac{1}{2}\right)^2 z^{-2} + \cdots$

$= \frac{\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}} = \frac{1}{2z-1} \quad |z| > \frac{1}{2}$

(2) $x_2(n) = \left(\frac{1}{2}\right)^n u(-1-n)$

$X_2(z) = \frac{-1}{1-\frac{1}{2}z^{-1}} \quad |z| < \frac{1}{2}$

$= \frac{2z}{1-2z} \quad |z| < \frac{1}{2}$

(3) $x_3(n) = \left(\frac{1}{2}\right)^{|n|}$

$X_3(z) = \sum_{n=-\infty}^{+\infty} \left(\frac{1}{2}\right)^{|n|} z^{-n}$

$= \sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^{-n} z^{-n} + \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n z^{-n}$

$= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n z^{n} + \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n z^{-n}$

$= \frac{\frac{1}{2}z}{1-\frac{1}{2}z} + \frac{1}{1-\frac{1}{2}z^{-1}} \quad \left(\frac{1}{2} < z < 2\right)$

$= -\frac{z}{z-2} + \frac{z}{z-\frac{1}{2}}$

6. $X(z) = \frac{1-z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$

$X(z)=\dfrac{\frac{1}{3}}{1-\frac{1}{2}z^{-1}}+\dfrac{\frac{2}{3}}{1-2z^{-1}}$

① $|z|>2$ 时 $x(n)=\frac{1}{3}(\frac{1}{2})^n u(n)+\frac{2}{3}2^n u(n)$

② $|z|<\frac{1}{2}$ 时 $x(n)=-\frac{1}{3}(\frac{1}{2})^n u(-n-1)-\frac{2}{3}2^n u(-n-1)$

③ $\frac{1}{2}<|z|<2$ 时 $x(n)=\frac{1}{3}(\frac{1}{2})^n u(n)-\frac{2}{3}2^n u(-n-1)$

三 1. 由 $y_{zs}(t)=h(t)*x(t)$

$=\int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau$

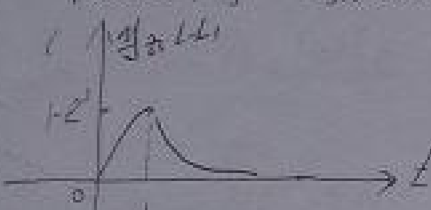
$=\int_0^1 e^{-(t-\tau)}u(t-\tau)d\tau$

$=\int_0^1 e^{\tau-t}u(t-\tau)d\tau$

$t<0$ 时 $y_{zs}(t)=0$

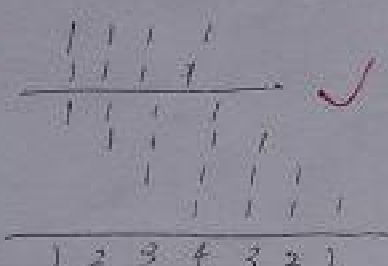
$1>t>0$ 时 $y_{zs}(t)=\int_0^t e^{\tau-t}d\tau=1-e^{-t}$

$\infty>t>1$ 时 $y_{zs}(t)=\int_0^1 e^{\tau-t}d\tau=e^{1-t}-e^{-t}$



2. $y_{zs}(n)=h(n)*x(n)$

利用表法原理



确定首项为 k:

$y_{zs}(1)=1$

$\therefore y_{zs}(n)=\{1,2,3,4,3,2,1\}$

四 1. $\frac{1}{2}\cos t$ · $H(j\Omega)=\delta(\Omega+1)+\delta(\Omega-1)$

$y(t)=\frac{1}{2\pi}(\cos t)=\frac{\cos t}{2\pi}$

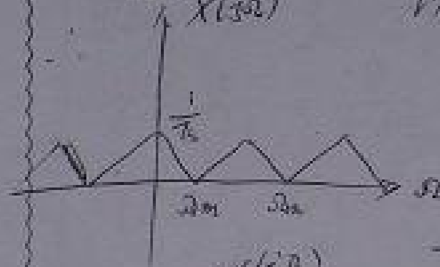
2. 采样: $\frac{1}{T_s}\geq 4k$ $T_s\leq\frac{1}{4000}s$

$x_s(t)=x(t)\cdot\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$

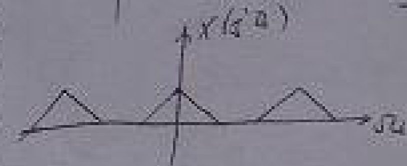
$X_s(j\Omega)=\frac{1}{2\pi}X(j\Omega)*\frac{2\pi}{T_s}\sum_{n=-\infty}^{\infty}\delta(\Omega-\frac{n2\pi}{T_s})$

$=\frac{1}{T_s}\sum_{n=-\infty}^{\infty}X[j(\Omega-\frac{n2\pi}{T_s})]$

以 $T_s=\frac{1}{4000}$ 为例



$T_s>\frac{1}{4000}$ 时



三 $i(t)\cdot 3+\frac{di(t)}{dt}+\frac{1}{2}\int_{-\infty}^{t}i(t)dt=e(t)$

两边同时微分:

$3i'(t)+i''(t)+2i(t)=e'(t)$

$i''(t)+3i'(t)+2i(t)=e'(t)$

① 方程两边同时取 S 变换

$s^2I(s)-si(0)+3sI(s)-3i(0)+2I(s)$

$=sE(s)=1$

$I(s)(s^2+3s+2)=\underline{si(0)+3i(0)+1}$

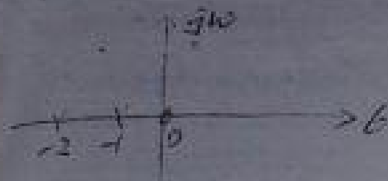
$I(s)=\dfrac{si(0)+3i(0)}{s^2+3s+2}+\dfrac{1}{s^2+3s+2}$

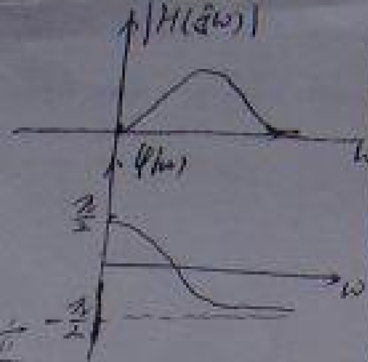
零输入响应: $L^{-1}\left\{\frac{s+3}{s^2+3s+2}\right\}=L^{-1}\left\{\frac{2}{s+1}+\frac{-1}{s+2}\right\}$

$=2e^{-t}u(t)-e^{-2t}u(t)=(2e^{-t}-e^{-2t})u(t)$

零状态响应: $L^{-1}\left\{\frac{1}{s+1}-\frac{1}{s+2}\right\}=(e^{-t}-e^{-2t})u(t)$

② $H(s)=\dfrac{s}{s^2+3s+2}=\dfrac{s}{(s+1)(s+2)}$





④ 因为系统的极点都在左半面左半平面，∴系统是稳定的。又由上面的频率特性曲线知 $|H(j\omega)| \neq C$，$\varphi(\omega)$ 不是 ω 的线性函数，∴不满足不失真传输条件。

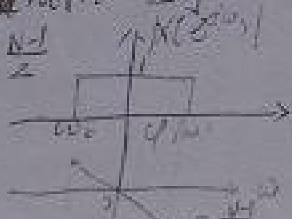
五. 1. $x(n) = \frac{W_c}{\pi} Sa[W_c(n - \frac{N-1}{2})]$

$u(\omega+W_c) - u(\omega-W_c) \xleftrightarrow{IDTFT} \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega n} d\omega = \frac{1}{2\pi}\int_{-W_c}^{W_c}$

$\frac{e^{j\omega n}}{jn} = \frac{1}{2j\pi n}[e^{jnW_c} - e^{j(-nW_c)}] = \frac{\sin nW_c}{\pi n}$

$= \frac{W_c}{\pi} Sa(nW_c)$　∴ $\frac{W_c}{\pi} Sa[W_c(n - \frac{N-1}{2})] \xleftrightarrow{DTFT}$

$[u(\omega+W_c) - u(\omega-W_c)] e^{-j\omega\frac{N-1}{2}}$



$y(n) = x(n)x(n)$

$DTFT[y(n)] = \frac{1}{2\pi} X(e^{j\omega}) * X(e^{j\omega})$

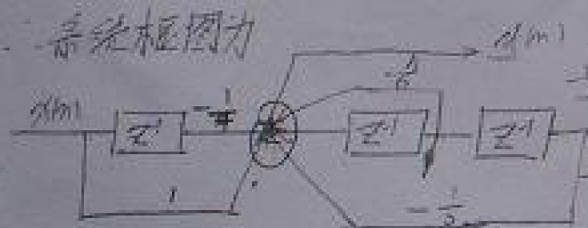
$= \frac{1}{2\pi}\int_{-\pi}^{\pi} [u(\theta+W_c) - u(\theta-W_c)] e^{-j\theta\frac{N-1}{2}} \cdot [u(\omega-\theta+W_c) - u(\omega-\theta-W_c)]$

$e^{-j(\omega-\theta)\frac{N-1}{2}} d\theta \; \frac{1}{2\pi} e^{-j\omega\frac{N-1}{2}} \int_{-W_c}^{W_c} [u(\omega-\theta+W_c) - u(\omega-\theta-W_c)] d\theta$

2.

① $Y(z) = X(z) - \frac{1}{4} z^{-1} X(z) + \frac{5}{6} z^{-1} Y(z) - \frac{1}{6} z^{-2} Y(z)$

∴系统框图为



② $H(z) = \frac{1 - \frac{1}{4}z^{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}$

$= \frac{(1 - \frac{1}{4}z^{-1})}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{3}z^{-1})}$

$= \frac{\frac{3}{2}}{1 - \frac{1}{2}z^{-1}} + \frac{-\frac{1}{2}}{1 - \frac{1}{3}z^{-1}}$

$h(n) = [\frac{3}{2}(\frac{1}{2})^n - \frac{1}{2}(\frac{1}{3})^n] u(n)$

③ 差分方程两边同时取Z变换

$Y(z) - \frac{5}{6}z^{-1}[Y(z) + z\,y(-1)] + \frac{1}{6}z^{-2}$

$[Y(z) + y(-1)z^{-1} + y(-2)z^{-2}] = \frac{1}{1 - \frac{1}{4}z^{-1}}$

$-\frac{1}{4}z^{-1}\frac{1}{1-\frac{1}{4}z^{-1}} = 1$

$Y(z) = \frac{\frac{5}{6}y(-1) - \frac{1}{6}z^{-1}y(-1) + 1}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}$

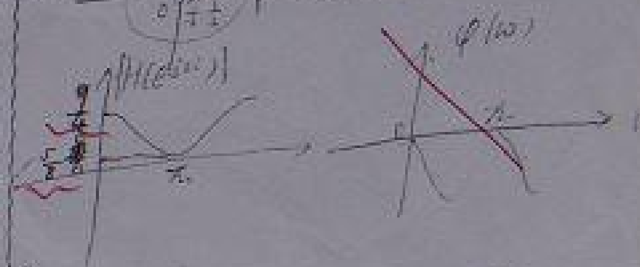
零输入响应：$y_{zi} = Z^{-1}\{\frac{\frac{5}{6} - \frac{1}{6}z^{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}\}$

$= Z^{-1}\{\frac{\frac{3}{2}}{1 - \frac{1}{2}z^{-1}} + \frac{-\frac{2}{3}}{1 - \frac{1}{3}z^{-1}}\}$

$= [\frac{3}{2}(\frac{1}{2})^n - \frac{2}{3}(\frac{1}{3})^n] u(n)$

$H(z) = \frac{z^2 - \frac{1}{4}z}{z^2 - \frac{5}{6}z + \frac{1}{6}}$

④



六. $H(s) = \frac{s+3}{s^2+3s+2} = \frac{s+3}{(s+2)(s+1)} = \frac{-1}{s+2}$

$+ \frac{2}{s+1} = \frac{-s^{-1}}{1+2s^{-1}} + \frac{2s^{-1}}{1+s^{-1}}$

$y(t) = -\lambda_1(t) + 2\lambda_2(t)$

$\lambda_1'(t) = -2\lambda_1(t) + f(t)$

$\lambda_2'(t) = -\lambda_2(t) + f(t)$

整理 $\begin{bmatrix}\lambda_1'(t)\\ \lambda_2'(t)\end{bmatrix} = \begin{bmatrix}-2 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}\lambda_1(t)\\ \lambda_2(t)\end{bmatrix} + \begin{bmatrix}1\\ 1\end{bmatrix} f(t)$

$y(t) = \begin{bmatrix}-1 & 2\end{bmatrix}\begin{bmatrix}\lambda_1(t) & \lambda_2(t)\end{bmatrix}$

$A = \begin{bmatrix}-2 & 0\\ 0 & -1\end{bmatrix}$ 下面求过渡矩阵：

$\Phi(t) = L^{-1}\{[sI - A]^{-1}\} = L^{-1}\{\frac{1}{(s+2)(s+1)}\begin{bmatrix}s+1 & 0\\ 0 & s+2\end{bmatrix}\}$

$= \begin{bmatrix}e^{-2t} & 0\\ 0 & e^{-t}\end{bmatrix} u(t)$