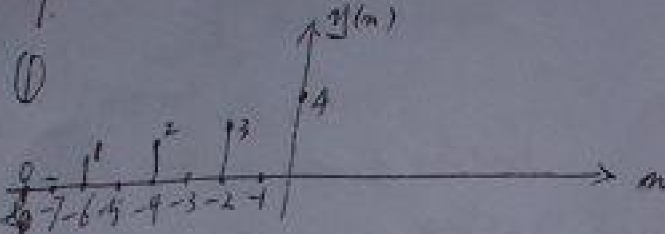
2008.

一.

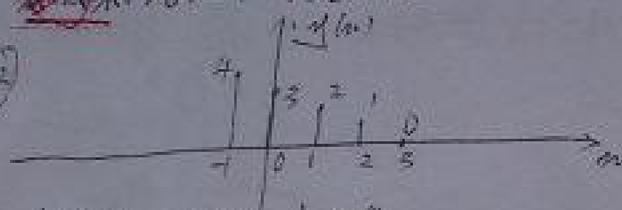
1.

①



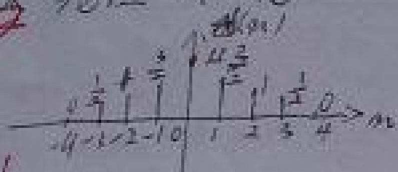
因果. 线性, 时变系统

②



非因果 线性 时不变

③



因果, 线性, 时变

① $H_1(s)=s$

$Y_1(s)=X(s)H_1(s)=1$

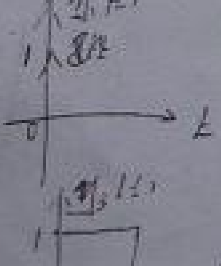
$y_1(t)=\delta(t)$

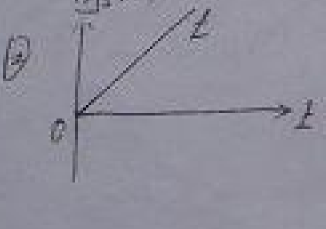
② $H_2(s)=\frac{1}{s}$ $Y_2(s)=\frac{1}{s^2}$

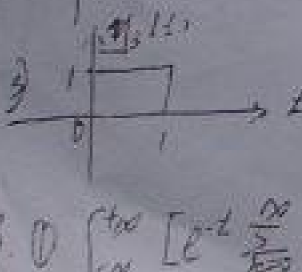
$y_2(t)=tu(t)$

$H_3(s)=1-e^{-s}$ $Y_3(s)=\frac{1-e^{-s}}{s}$

$y_3(t)=u(t)-u(t-1)$







3. ① $\int_{-\infty}^{+\infty}[e^{-t}\sum_{k=0}^{\infty}\delta(t-k)]dt=1+e^{-1}+e^{-2}$

$+\cdots e^{-n}+\cdots=\frac{1}{1-e^{-1}}$

② $\int_{-1}^{4}[\cos t\cdot\sum_{k=-\infty}^{+\infty}\delta(t-k\frac{\pi}{2})]dt=\cos 0+\cos\frac{\pi}{2}$

$+\cos\frac{2\pi}{2}+\cos\frac{3\pi}{2}+\cos 2\pi+\cos\frac{5\pi}{2}=1$

③ $\int_{0}^{1}e^{-j\pi t}[\delta(t-\frac{1}{2})+\delta(t+\frac{1}{2})]dt=e^{-j\pi\frac{1}{2}}$

$=\cancel{2\cos\frac{\pi}{2}=0}=e^{-j\frac{\pi}{2}}=-j$

④ $\int_{-\infty}^{+\infty}e^{-\tau}\delta(t-\tau+k)d\tau=\int_{-\infty}^{+\infty}e^{-\tau}\delta(\tau-t-k)d\tau$

$=e^{-t-k}$

二.

1.

① $X(j\Omega)=\int_{-\infty}^{+\infty}x(t)e^{-j\Omega t}dt$

$F[x(t-t_0)]=\int_{-\infty}^{+\infty}x(t-t_0)e^{-j\Omega t}dt$ 令 $\tau=t-t_0$

$F\{x(t-t_0)\}=\int_{-\infty}^{+\infty}x(\tau)e^{-j\Omega(\tau+t_0)}d\tau$

$=X(j\Omega)\cdot e^{-j\Omega t_0}$

② $\frac{dX(j\Omega)}{d\Omega}=\frac{d[\int_{-\infty}^{+\infty}x(t)e^{-j\Omega t}dt]}{d\Omega}=\int_{-\infty}^{\infty}\frac{d[x(t)e^{-j\Omega t}]}{d\Omega}dt$

$=\int_{-\infty}^{+\infty}-jt\,x(t)e^{-j\Omega t}dt$

$\therefore F\{tx(t)\}=j\frac{dX(j\Omega)}{d\Omega}$

2.

① $X(s)=\int_{0}^{+\infty}x(t)e^{-st}dt$

$\int_{0}^{+\infty}x'(t)e^{-st}dt=\int_{0}^{+\infty}e^{-st}dx(t)=x(t)e^{-st}\Big|_{0}^{+\infty}$

$-\int_{0}^{+\infty}-s\,x(t)e^{-st}dt=-x(0^-)+sX(s)$

得证

② $\int_{0}^{+\infty}x(t)e^{s_0 t}e^{-st}dt=\int_{0}^{+\infty}x(t)e^{-t(s-s_0)}dt$

$=X(s-s_0)$

3. $X(e^{j\omega})=\sum_{n=-\infty}^{\infty}x(n)[\cos n\omega-j\sin n\omega]$

$Re\{X(e^{j\omega})\}=\sum_{n=-\infty}^{\infty}x(n)\cos n\omega$

$Im\{X(e^{-j\omega})\}=\sum_{n=-\infty}^{\infty}x(n)\sin n\omega$

因为 $x(n)$ 为实序列 $\cos n\omega$ 为 ω 的偶函数 $\sin n\omega$

三.

1.

$y_{zs}(t)=x(t)*h(t)$

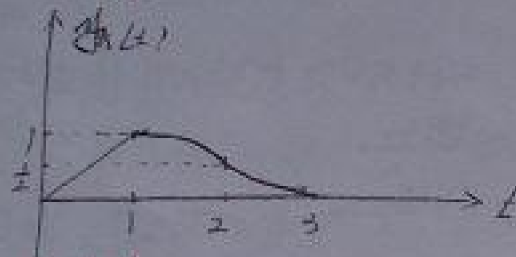
$=\int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$

$t<0$ 时 $y_{zs}(t)=0$

$1>t\geq 0$ 时 $y_{zs}(t)=-\frac{1}{2}t^2+2t$

$2>t\geq 1$ 时 $y_{zs}(t)=1$ ~~…~~ $-t+\frac{5}{2}$

$3>t\geq 2$ 时 $y_{zs}(t)=$ ~~…~~ $\frac{1}{2}t^2-3t+4\frac{1}{2}$



(修改技术)

2. $y(n)=x(n)*h_1(n)*h_2(n)*h_2(n)$

$h(n)=$ ~~$\delta(n)*h_1(n)*h_2(n)*h_2(n)$~~ 卷积后 n_1+n_2-1

$=h_1(n)*h_2(n)*h_2(n)$

$=[\delta(n)+3\delta(n-1)+3\delta(n-2)+2\delta(n-3)+\delta(n-4)]$

$*[h_2(n)*h_2(n)]$ $5+x-1=7$ $x=3$

由题意 $h(n)$ 长度为 7 $h_1(n)$ 长度为 5

$\therefore h_2(n)*h_2(n)$ 长度为 3 $h_2(n)$ 长度为 2 $3=2x-1$

设 $X(n)=h_2(n)*h_2(n)$ 由因果性知

~~…~~

1 3 3 2 1

x_1 x_2

1 5 6 11 8 4 1

x_1 $3x_1$ $3x_1$ $2x_1$ x_1

x_2 $3x_2$ $3x_2$ $2x_2$ x_2

$x(0)=h_2(0)h_2(0)=h_2^2(0)$

$x(1)=2h_2(0)h_2(1)$

$x(2)=2h_2(1)h_2(1)$

所以 $h_2(0)=1$ $h_2(1)=1$

$h_2(n)=\delta(n)+\delta(n-1)$

四 1.

$y(t)=[x(t)s(t)]*h(t)$

$X(j\Omega)=2\pi\delta(\Omega)+\pi[\delta(\Omega+1)+\delta(\Omega-1)]$

$+\pi[\delta(\Omega+2)+\delta(\Omega-2)]$

$S(j\Omega)=\pi[\delta(\Omega+2)+\delta(\Omega-2)]$

$\frac{1}{2\pi}X(j\Omega)*S(j\Omega)=\frac{\pi}{2}[2\delta(\Omega)+\delta(\Omega+1)+\delta(\Omega-1)$

$+\delta(\Omega+2)+\delta(\Omega-2)]*[\delta(\Omega+2)+\delta(\Omega-2)]$

$=\frac{\pi}{2}[2\delta(\Omega+2)+2\delta(\Omega-2)+\delta(\Omega+3)+\delta(\Omega-1)+\delta(\Omega+1)+\delta(\Omega-3)+\delta(\Omega)+\delta(\Omega+4)+\delta(\Omega)+\delta(\Omega-4)]$

$Y(j\Omega)=\frac{\pi}{2}[2\delta(\Omega)+\delta(\Omega+1)+\delta(\Omega-1)]$

$=\pi\delta(\Omega)+\frac{\pi}{2}[\delta(\Omega+1)+\delta(\Omega-1)]$

$y(t)=\frac{1}{2}+\frac{1}{2}\cos t$

2. $y(t)=x_1(t)x_2(t)\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$

$x(t)=x_1(t)\cdot x_2(t)\leftrightarrow\frac{1}{2\pi}X_1(j\Omega)*X_2(j\Omega)$

由 $x_1(t)$, $x_2(t)$ 的带限性知:

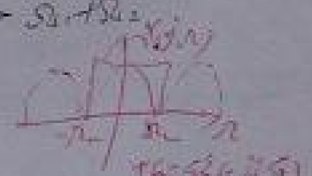
$X(j\Omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X_1(j\Omega')X_2[j(\Omega-\Omega')]d\Omega'$

当 $\Omega<-\Omega_1-\Omega_2$ 时 $X(j\Omega)=0$

$\Omega>\Omega_1+\Omega_2$ 时 $X(j\Omega)=0$

$\therefore |X(j\Omega)|=0$ $|\Omega|>\Omega_1+\Omega_2$

$\therefore T_s=\frac{\pi}{\Omega_1+\Omega_2}$



(相移特性说明)

3. $y(t)=[x(t)\sin\Omega_m t\cos\Omega_m t]*h(t)$

$F[x(t)\cdot\sin\Omega_m t\cos\Omega_m t]=\frac{1}{2}F[x(t)\sin 2\Omega_m t]$

$=\frac{1}{2}[X(j\Omega+j2\Omega_m)-X(j\Omega-j2\Omega_m)]\cdot\frac{1}{2j}$

$Y(j\Omega)=\frac{1}{4j}[X(j\Omega+j2\Omega_m)-X(j\Omega-j2\Omega_m)]\cdot H(j\Omega)$

$=\frac{1}{4j}[X(j(\Omega+2\Omega_m))-X(j(\Omega-2\Omega_m))]$ $|\Omega|<\Omega_m$

$=0$

$\therefore$ 输入信号为: $\frac{1}{2}x(t)\sin 2\Omega_m t$ 输出: 0

四

①

$y(t)+y'(t)+y''(t)=x(t)$

② $\because x(t)=\delta''(t)+\delta'(t)+\delta(t)$ 利用冲激函数匹配法求 y_{zs}

可设 $y''(t)=a\delta''(t)+b\delta'(t)+c\delta(t)$

$y'(t)=a\delta'(t)+b\delta(t)+c\varepsilon(t)$

$y(t)=a\delta(t)+b\varepsilon(t)$

得 $a=1$ $b=c=0$

$\therefore y_{zs}(t)=\delta(t)$

$\therefore y_{zs}'(0^+)=c+y_{zs}'(0^-)=0+0=0$

$y_{zs}(0^+)=b+y_{zs}(0^-)=0+0=0$

$\therefore y_{zs}=e^{-\frac{1}{2}t}[\cos\frac{\sqrt3}{2}t+\sin\frac{\sqrt3}{2}t]$

③ 由题意 特征方程为 $\lambda^2+\lambda+1=0$

特征根：$\lambda_1=\frac{-1+\sqrt3 j}{2}$ $\lambda_2=\frac{-1-\sqrt3 j}{2}$

$\therefore$ 可设 $y_{zi}(t)=Ae^{\frac{-1+\sqrt3 j}{2}t}+Be^{\frac{-1-\sqrt3 j}{2}t}$

$$\begin{cases}A+B=1\\ A(\frac{-1+\sqrt3 j}{2})+B(\frac{-1-\sqrt3 j}{2})=-\frac{1}{2}\end{cases}\quad\therefore A=\frac{1}{2},\ B=1/2$$

$\therefore y_{zi}(t)=\frac{1}{2}e^{-\frac{1}{2}t}\,2\cos\frac{\sqrt3}{2}t=e^{-\frac{1}{2}t}\cos\frac{\sqrt3}{2}t$ $t>0$

④ $x(t)=\sin t$ $X(s)=\frac{1}{s^2+1}$

$\therefore Y(s)=X(s)H(s)$

$=\frac{1}{s^2+1}\cdot\frac{1}{s^2+s+1}=\frac{s}{s^2+1}+\frac{-s}{(s+\frac{1}{2})^2+\frac{3}{4}}$

$=\frac{s}{s^2+1}-\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}}+\frac{2}{\sqrt3}\cdot\frac{1}{2}\frac{\frac{\sqrt3}{2}}{(s+\frac{1}{2})^2+\frac{3}{4}}$

$y(t)=\cos t-e^{-\frac{1}{2}t}\cos\frac{\sqrt3}{2}t+\frac{1}{\sqrt3}\sin\frac{\sqrt3}{2}t\,\varepsilon(t)$

稳态响应 $y(t)=\cos t\ \varepsilon(t)$

⑤ 由于 $H(s)$ 两个极点均在 S 左半平面，$\therefore$ 系统稳定

$|H(j\omega)|=\left|\frac{-\omega^2}{1-\omega^2+j\omega+1}\right|=\left|\frac{-\omega^2(1-\omega^2-j\omega)}{(1-\omega^2)^2+\omega^2}\right|$ 是 ω 的函数

$\therefore$ 系统不满足不失真传输条件

五.

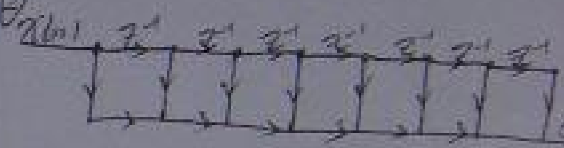
① $H(z)=\frac{1-z^{-8}}{1-z^{-1}}$

$=(1+z^{-1})(1+z^{-2})(1+z^{-4})$

$=1+z^{-1}+z^{-2}+z^{-3}+z^{-4}+z^{-5}+z^{-6}+z^{-7}$

$\therefore h(n)=\delta(n)+\delta(n-1)+\delta(n-2)+\delta(n-3)+\delta(n-4)+\delta(n-5)+\delta(n-6)+\delta(n-7)$

②



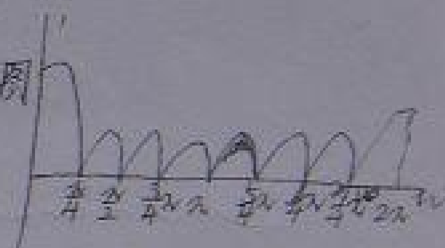
③ $H(z)=\frac{(z^4+1)(z^2+1)(z+1)}{z^7}$

极点 $z=0$ (7阶)

零点 $e^{j\frac{\pi}{4}n}$ $n=1\ 2\ 3\ 4\ 5\ 6\ 7$

④ 用幅频特性图

表示：



⑤ $x(n)=\cos\frac{\pi}{4}n$

$H(e^{j\theta})=\frac{1-e^{-j8\theta}}{1-e^{-j\theta}}\Big|_{z=e^{j\theta}}$ $\theta=\frac{\pi}{4}$

$H(e^{j\theta})\Big|_{\theta=\frac{\pi}{4}}=\frac{1-e^{-j2\pi}}{1-e^{-j\frac{\pi}{4}}}=\frac{0}{1-\frac{\sqrt2}{2}+j\frac{\sqrt2}{2}}=0$

$y(n)=x(n)*h(n)$ 又 $\cos n=-\cos(n+2)=-\cos(n-2)$

$\therefore y(n)=0$

$\therefore$ 稳态响应为 0

2.

$y(n) - \frac{10}{3} y(n-1) + y(n-2) = 2x(n) - \frac{10}{3} x(n-1)$

$H(z) = \frac{2 - \frac{10}{3} z^{-1}}{1 - \frac{10}{3} z^{-1} + z^{-2}} = \frac{2 - \frac{10}{3} z^{-1}}{(1 - \frac{1}{3} z^{-1})(1 - 3 z^{-1})}$

$= \frac{1}{1 - \frac{1}{3} z^{-1}} + \frac{1}{1 - 3 z^{-1}}$

①系统因果的

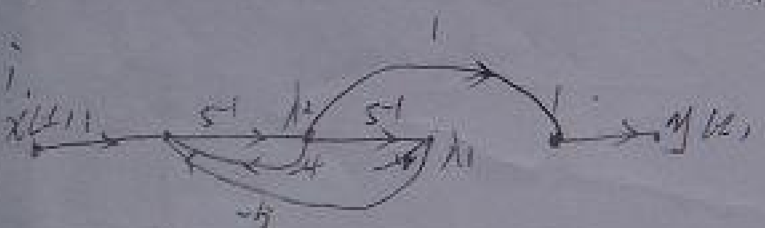
$h_1(n) = [(\frac{1}{3})^n + 3^n] u(n)$ $\quad |z| > 3$

②非因果稳定的:

$h_2(n) = [(\frac{1}{3})^n u(n) - 3^n u(-n-1)]$ $\quad 3 > |z| > \frac{1}{3}$

③非因果不稳定

$h_3(n) = [-(\frac{1}{3})^n u(-n-1) - 3^n u(-n-1)]$ $\quad \cancel{|z| < 3}$ $\quad |z| < \frac{1}{3}$



$\begin{cases} \lambda_1'(t) = \lambda_2(t) \\ \lambda_2'(t) = x(t) + (-5)\lambda_1(t) - 4\lambda_2(t) \end{cases}$

$y(t) = \lambda_2(t)$

$\begin{bmatrix} \lambda_1'(t) \\ \lambda_2'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -4 \end{bmatrix} \begin{bmatrix} \lambda_1(t) \\ \lambda_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x(t)$

$y(t) = [0 \ \ 1] \begin{bmatrix} \lambda_1(t) \\ \lambda_2(t) \end{bmatrix}$

$\Phi = (sI - A)^{-1} = \begin{bmatrix} s & -1 \\ 5 & s+4 \end{bmatrix}^{-1} = \frac{1}{s^2+4s+5} \begin{bmatrix} s+4 & 1 \\ -5 & s \end{bmatrix}$