北京化工大学
攻读硕士学位研究生入学考试
《数学专业综合》样题

注意事项

1. 答案必须写在答题纸上，写在试卷上均不给分。
2. 答题时可不抄题，但必须写清题号。
3. 答题必须用蓝、黑墨水笔或圆珠笔，用红色笔或铅笔均不给分。

一、（11 分）对于微分方程

$$y=(\frac{\mathrm{d}y}{\mathrm{d}x})^2-x\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{x^2}{2}$$

1.（8 分）给出所有解。

2.（3 分）作出过 $(0,0)$ 的积分曲线图。

二、（10 分）对于微分方程初值问题 $\begin{cases}\dfrac{dy}{dx}=f(x,y)\\ y(x_0)=y_0\end{cases}$，设 $f(x,y)$ 在区域 $G=[x_0,+\infty)\times(-\infty,+\infty)$ 上连续，有界，关于 y 满足局部的 Lipschitz 条件，证明此初值问题在 $[x_0,+\infty)$ 上存在唯一的解。

三、(总分 14，每小题 7 分)

1. 求微分方程

$$y''+4y'+4y=\cos 2x$$ 的通解。

2. 求微分方程组 $\begin{cases}\dfrac{\mathrm{d}x}{\mathrm{d}t}=2x+3y\\ \dfrac{\mathrm{d}y}{\mathrm{d}t}=3x+2y\end{cases}$ 的通解。

四、(13) 设 E 为 Lebesgue 可测集，如果$\{u_n\}$在$L^2(E)$中收敛到u，

是否存在子序列$\{u_{n_k}\}\subset\{u_n\}$使得$\lim_{k\to\infty}u_{n_k}(x)=u(x), a.e. x\in E$，

为什么?

五、(12) 计算

$$\lim_{n\to\infty}\int_{(0,\infty)}\frac{dt}{(1+\frac{t}{n})^n t^{1/n}}。$$

六、(10 分): 给定一条空间曲线 $\begin{cases}x=t\\ y=t^2\\ z=t^3\end{cases}$，$t\in R$ 求它的切线面的参数方程并且证明它的切线是切线面上的曲率线.

七、10 分) 证明如果一条空间曲线的法平面总经过一个定点, 则该曲线位于一个球面上.

八、(20 分）专业英语：将下列英文翻译成中文

题目： Continuous Functions of One Real Variable

The concept of continuity is one of the most important and also one of the most fascinating ideas in all of mathematics. Before we give a precise technical definition of continuity, we shall briefly discuss the concept in an informal and intuitive way to give the reader a feeling for its meaning.

Roughly speaking the situation is this: Suppose a function *f* has the value *f* (*p*) at a certain point *p*. Then *f* is said to be continuous at *p* if at every nearby point *x* the function value *f* (*x*) is close to *f* (*p*). Another way of putting it is as follows: If we let *x* move toward *p*, we want the corresponding function value *f* (*x*) to become arbitrarily close to *f* (*p*), regardless of the manner in which *x* approaches *p*. We do not want sudden jumps in the values of a continuous function.

Consider the graph of the function *f* defined by the equation $f(x) = x-[x]$, where $[x]$ denotes the greatest integer $< x$. At each integer we have what is known a jump discontinuity. For example, $f(2) = 0$,but as x approaches 2 from the left, $f(x)$ approaches the value 1, which is not equal to $f(2)$.Therefore we have a discontinuity at 2. Note that $f(x)$ does approach $f(2)$ if we let x approach 2 from the right, but this by itself is not enough to establish continuity at 2. In case like this, the function is called continuous from the right at 2 and discontinuous from the left at 2. Continuity at a point requires both continuity from the left and from the right.

In the early development of calculus almost all functions that were dealt with were continuous and there was no real need at that time for a penetrating look into the exact meaning of continuity. It was not until late in the 18th century that discontinuous functions began appearing in connection with various kinds of physical problems. In particular, the work of J.B.J. Fourier (1758-1830) on the theory of heat forced mathematicians in the early 19th century to examine more carefully the exact meaning of the word "continuity".

A satisfactory mathematical definition of continuity, expressed entirely in terms of properties of the real number system, was first formulated in 1821 by the French mathematician, Augustin-Louis Cauchy (1789-1857). His definition, which is still used today, is most easily explained in terms of the limit concept to which we turn now.

The definition of the limit of a function. (omitted)

The definition of continuity of a function.

In the definition of limit we made no assertion about the behavior of *f* at the point p itself. Moreover, even if *f* is defined at p, its value there need not be equal to the limit A. However, if it happens that *f* is defined at p and if it also happens that $f(p) = A$, then we say the function *f* is continuous at p. In other words, we have the following definition.

Definition of continuity of a function at a point.

A function f is said to be continuous at a point p if

(a) f is defined at p, and (b) $f(x) \to A$ as $x \to p$

This definition can also be formulated in term of neighborhoods. A function f is continuous at p if for every neighborhood $N_1(f(p))$ there is a neighborhood $N_2(p)$ such that

$$f(x) \in N_1(f(p)) \text{ whenever } x \in N_2(p).$$

In the ε-δ terminology, where we specify the radii of the neighborhoods, the definition of continuity can be restated ad follows:

Function f is continuous at p if for every $\varepsilon > 0$, there is a $\delta > 0$ such that

$$|f(x) - f(p)| < \varepsilon \text{ whenever } |x - p| < \delta$$

(omitted)

Vocabulary

intuitive 直观的

jump discontinuity 跳跃不连续

formulate 用公式表示,阐述

terminology 术语学