2001 年西北工业大学硕士研究生入学考试
自动控制原理试题简要解答

1. (1) $K = 3$, $\omega = \sqrt{2}$

   (2) $2 \leqslant K < 3$

2. (1) $d = 5$, $\begin{cases} \omega = 0 \\ K^* = 1 \end{cases}$, $\begin{cases} \omega = 5 \\ K^* = \dfrac{4}{5} \end{cases}$

   (2) $1 < K = K^* < \infty$

   (3) $\lambda_{1,2} = -\dfrac{5\sqrt{2}}{2} \pm z\dfrac{5\sqrt{2}}{2}$, $K = 0.317$

   $\sigma = 4.32\%$, $t_s = 0.99\ \text{s}$

3. (1) $d = \sqrt{10}$

   (2) $K_t = 0.216$, $\sigma = 16\%$, $t_s = 2.216\ \text{s}$

4. (1) $\Phi(s) = \dfrac{K_\varphi \omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \dfrac{0.875 \times 4^2}{s^2 + 2 \times 0.375 \times 4s + 4^2} =$

   $\dfrac{14}{s^2 + 3s + 16}$

   (2) $\sigma = 28\%$, $t_s = 2.3\ \text{s}$

   (3) $\omega_c = 3.74$, $\gamma = 43.1^\circ$

5. (1) $\omega_c = 1$, $\gamma = 5.71^\circ$, $h = \infty$

   (2) $\Phi(s) = \dfrac{1}{s^2 + 0.1s + 1}$

   (3) $\omega_r = 0.9975$, $M_r = 10$

6. (1) $\omega_c = 5.3133$, $\gamma(5.3133) = -46^\circ$

若用一级超前校正，则 $\varphi_m = \gamma^* - \gamma + 5^\circ = 40^\circ + 46^\circ + 5^\circ = 91^\circ$（不可能达到）。若用滞后，则校正后系统在 $\omega'_c = 2.5$ 处的相角裕度为 $\gamma(2.5) = -9.06^\circ$（不满足要求）。

故只能采用滞后-超前校正。

(2) $\omega_c^* = 3$，$\gamma^* = 43.65^\circ$，满足要求

7. $5 \leqslant K < 20.343$

8. $K = \dfrac{K^*}{2} < \dfrac{3\pi a}{2}$

2002 年西北工业大学硕士研究生入学考试自动控制原理试题简要解答

1. (1) $\dfrac{C(s)}{E(s)} = \dfrac{2G_1G_2 + G_2 - G_1}{1 + G_1G_2}$

(2) $\dfrac{C(s)}{R(s)} = \dfrac{2G_1G_2 + G_2 - G_1}{1 + G_1G_2 + (2G_1G_2 + G_2 - G_1)H}$

(3) $0.5 < K_1 < 2$

2. (1) $d = -10$，$\theta_{p_1} = -150^\circ$，$\theta_{p_2} = 150^\circ$

(2) $0 < K_D < 0.1$

(3) $K_D = 0.0414$

$$\Phi(s) = \frac{100(1 + 0.0414s)}{s^2 + 14.14s + 100}$$

3. (1) $\Phi(s) = \dfrac{316.2}{s^2 + 10s + 316.2}$

(2) $\sigma = 40\%$，$t_s = 0.7\ \text{s}$

(3) $G(s) = \dfrac{316.2}{s\left(\frac{s}{10} + 1\right)}$

渐近线 $\lim\limits_{\omega \to 0^+} \operatorname{Re}[G(j\omega)] = -3.162$，开环幅相曲线（略）。

4. $G(s) = \dfrac{K}{(T_1s + 1)(T_2s + 1)} = \dfrac{19}{(4.17s + 1)(0.4116s + 1)}$

5. (1) $K_c = 0.316$，$T = 0.316$

(2) $e_{ss} = 6.33$

6. (1) $G(z)=\dfrac{K(1+cz^{-1})z}{1+(a-K)z^{-1}+(b-Kc)z^{-2}}$

(2) 略

7. $K=\sqrt{10}\pi$， $\tau=0.322$

8. $\begin{cases}\dfrac{\dot{c}^2}{45}+\dfrac{(c+1)^2}{9}=1\\ \dfrac{\dot{c}^2}{45}+\dfrac{(c-1)^2}{9}=1\end{cases}$

2003 年西北工业大学硕士研究生入学考试
自动控制原理试题简要解答

1. (1) $-4<a<36$

(2) $a=0$

(3) $\omega_c=1$，$\gamma=76^\circ$

2. (1) $\begin{cases}\sigma_a=-\dfrac{5}{3}\\ \varphi_a=\pm 60^\circ, 180^\circ\end{cases}$， $\begin{cases}d_1=-2.87\\ d_2=-0.46\end{cases}$， $\begin{cases}\omega=0, K^*=10\\ \omega=2, K^*=30\end{cases}$

(2) $\Phi_1(s)=\dfrac{3.936}{(s+2.87)^2(s-0.74)}$，$\Phi_2(s)=\dfrac{10.88}{(s+0.46)^2(s+4.07)}$

(3) $1<K=\dfrac{K^*}{10}<1.09$

3. (1) $G(s)=\dfrac{4K}{s^2+5s+4}$

(2) $K=8$

(3) $\omega_c=5.657$， $\gamma=45.3^\circ$

4. (1) $G(s)=\dfrac{10.89}{s(s+1.98)}$

(2) 解法一　用串联校正

$$G_c(s)=\frac{4.5(s+1.98)}{s+7},\ K=7$$

解法二　用测速反馈

$$G_c(s)=\frac{49}{s(s+7)},\ K=7$$

5. (1) $0<K<8.35$　(2) $e_{ss}=1$

6. (1) 系统会自振　(2) $|c|=17.95$，$\omega=1$

北工业大学 2004 年硕士研究生入学考试

自动控制原理试题解答

一、 (1) $\Phi_e(s)=\dfrac{E(s)}{R(s)}=\dfrac{1+\dfrac{K_0K_ts}{s(s+1)}-\dfrac{K_0G_c(s)}{s(s+1)}}{1+\dfrac{K_0K_t}{s+1}+\dfrac{K_0(s+2)}{s^2(s+1)}}=$

$$\frac{s[s(s+1)+K_0K_ts-K_0G_c(s)]}{s^2(s+1)+K_0K_ts^2+K_0(s+2)}$$

$$D(s)=s^3+(1+K_0K_t)s^2+K_0s+2K_0=0$$

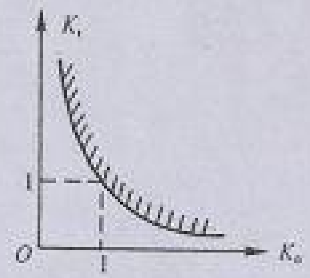

图 D.1.1

劳斯阵列

s^3 1 K_0

s^2 $1+K_0K_t$ $2K_0$ $\rightarrow K_0K_t>-1$

s^1 $\dfrac{K_0[1+K_0K_t-2]}{1+K_0K_t}$ $\rightarrow K_0K_t>1, K_t>\dfrac{1}{K_0}$

s^0 $2K_0$ $K_0>0$

(2) $e_{ss}=\lim\limits_{s\to 0}sR(s)\Phi_e(s)=\lim\limits_{s\to 0}s\dfrac{1}{s^2}\dfrac{s^2[s+1+K_0K_t-\frac{K_0}{s}G_c(s)]}{s^2(s+1)+K_0K_ts^2+K_0(s+2)}=$

$$\frac{1+K_0K_t-\frac{K_0}{s}G_c(s)}{2K_0}=0$$

故

$$G_c(s)=\frac{(1+K_0K_t)s}{K_0}$$

如图 D.1.1 所示。

二、(1) 绘制根轨迹，如图 D.1.2 所示。

$$G(s)=\frac{5K_0(s-1)}{(s^2+4s+4)(s+5)}\quad \begin{cases}K=\dfrac{K_0}{4}\\ \gamma=0\end{cases}$$

① 渐近线 $\begin{cases}\sigma_a=\dfrac{-2-2-5-1}{3-1}=-5\\ \varphi_a=\dfrac{(2K+1)\pi}{3-1}=\pm 90^\circ\end{cases}$

② 分离点 $\dfrac{2}{d+2}+\dfrac{1}{d+5}=\dfrac{1}{d-1}$

整理有 $d^2+d-11=0$

解得 $\begin{cases}d_1=-3.854\\ d_2=+2.854\end{cases}$

③ 与虚轴交点

$D(s)=(s^2+4s+4)(s+5)+5K_0(s-1)=s^3+9s^2+$
$(24+5K_0)s+(20-5K_0)=0$

$$\begin{cases}\text{Im}[D(j\omega)]=-\omega^3+(24+5K_0)\omega=0\\ \text{Re}[D(j\omega)]=-9\omega^2+(20-5K_0)=0\end{cases}$$

解得 $\begin{cases}\omega=0\\ K_0=\dfrac{20}{5}=4\end{cases}$

依根轨迹得 $0<K_0<4,\quad 0<K=\dfrac{K_0}{4}<1$

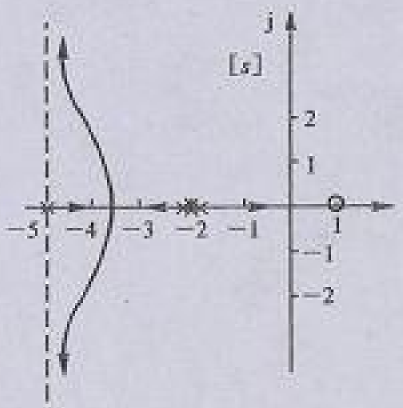

图 D.1.2

$$\frac{D(s)}{s+1}=s^2+8s+18,\quad \Phi(s)=\frac{0.4(s-1)(s+5)}{(s+1)(s^2+8s+18)}$$

三、(1) 依题意有

$$G(s)=\frac{K\left(\frac{s}{0.2}+1\right)}{s(s^2+2\xi s+1)\left(\frac{s}{20}+1\right)}$$

$$K=2,\quad 20\lg M_r=2.7\ \text{dB}$$

$$M_r=10^{\frac{2.7}{20}}=1.3646=\frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$\xi^4-\xi^2+0.1342=0,\quad \xi^2=\frac{1\pm\sqrt{1-4\times 0.134^2}}{2}=0.16,\quad \xi=0.4$$

故

$$G(s)=\frac{2\left(\frac{s}{0.2}+1\right)}{s(s^2+0.8s+1)\left(\frac{s}{20}+1\right)}$$

(2) 依题意有 $1<\omega_c<20$

$$G(j\omega_c)=\frac{2\times\frac{\omega_c}{0.2}}{\omega_c\times\omega_c^2\times 1}=\frac{10}{\omega_c^2}=1$$

$$\omega_c=\sqrt{10}=3.16$$

$$\gamma=180^\circ+\arctan\frac{3.16}{0.2}-90^\circ-\arctan\frac{0.8\times 3.16}{1-3.16^2}-\arctan\frac{3.66}{20}=180^\circ+86.38^\circ-90^\circ-$$
$$(180^\circ-15.7^\circ)-8.98^\circ=3.1^\circ$$

四、 $G_0(s)=\dfrac{K}{s\left(\frac{s}{10}+1\right)}\left(\dfrac{s}{100}+1\right)$

当 $K=K_c=100$ 时，作 $L_0(\omega)$ 如图 D.1.3 所示。

$$\omega_{c0}=\omega_{g0}=\sqrt{10\times 100}=31.6$$

系统临界稳定，故 K 的稳定范围为

$$0<K<100$$

(2) $G(s)=G_c(s)G_0(s)=\dfrac{100\left(\frac{s}{20}+1\right)}{s\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)\left(\frac{s}{200}+1\right)}$

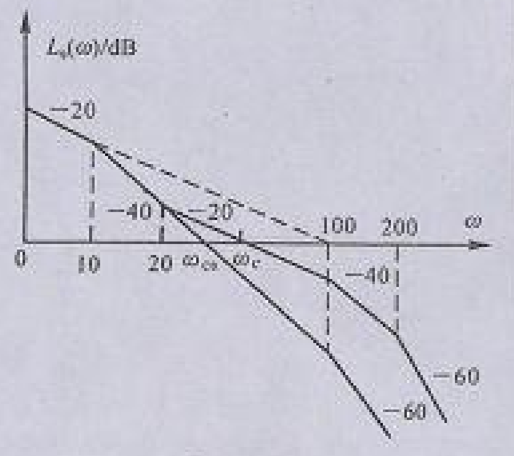

图 D.1.3

此时 $\omega_c^*=\dfrac{\omega_{c0}^2}{20}=\dfrac{31.6^2}{20}=50$

$$\gamma^*=180^\circ+\arctan\frac{50}{20}-90^\circ-\arctan\frac{50}{10}-\arctan\frac{50}{100}-\arctan\frac{50}{200}=$$
$$180^\circ-68.2^\circ-90^\circ-76.7^\circ-26.6^\circ-14^\circ 38.9^\circ>30^\circ$$

五、 $G_0(z)=(1-z^{-1})Kz^{-1}\mathscr{Z}\left[\dfrac{1/T_0}{s\left(s+\frac{1}{T_0}\right)}\right]=\dfrac{z-1}{z^2}K\mathscr{Z}\left[\dfrac{1}{s}-\dfrac{1}{s+\frac{1}{T_0}}\right]=$

$$\frac{z-1}{z^2}K\left[\frac{z}{z-1}-\frac{z}{z-e^{-T/T_0}}\right]=\frac{K}{z}\left[1-\frac{z-1}{z-e^{-T/T_0}}\right]=$$

$$\frac{K(1-e^{-T/T_0})}{z(z-e^{-T/T_0})}\ \xrightarrow{e^{-T/T_0}=0.2}\ \frac{0.8K}{z(z-0.2)}$$

(1) $D_\Phi(z)=z(z-0.2)+0.8K=z^2-0.2z+0.8K=0$

$$D_\Phi(1)=1-0.2+0.8K>0\longrightarrow K>-1$$

$$D_\Phi(-1)=1+0.2+0.8K>0\longrightarrow K>-1.5$$

$$0.8K<1\longrightarrow K<\frac{1}{0.8}=1.25$$

考虑题中条件：$K>0$，有 $0<K<1.25$

$$G(z)=D(z)G_0(z)=\frac{bz+c}{z-1}\frac{0.8}{z(z-0.2)}=\frac{0.8(bz+c)}{z(z-0.2)(z-1)}$$

$$D_\varphi(z)=z(z-0.2)(z-1)+0.8(bz+c)=z^3-1.2z^2+(0.2+0.8b)z+0.8c=0$$

又有
$$(z-a)^3=z^3-3az^2+3a^2z-a^3=0$$

比较系数
$$\begin{cases}3a=1.2\\3a^2=0.2+0.8b\\a^3=-0.8c\end{cases}$$

解出
$$a=0.4,\quad b=0.35,\quad c=-0.08$$

六、自振条件：
$$N(A)G(j\omega)=\frac{4}{\pi A}\frac{Ke^{-\tau s}\times 10}{j\omega(2+j\omega)^2}=-1$$

(1) $\tau=0$ 时
$$\frac{40K}{\pi A}=j\omega(2+j\omega)^2=4\omega^2-j\omega(4-\omega^2)$$

比较虚实部
$$\omega=2$$
$$\frac{40K}{\pi A}=4\omega^2$$
$$K=0.1\times\omega^2\pi A\xrightarrow{A=10Ac}2^2\pi\times\frac{1}{4}=\pi$$

(2)
$$\frac{40Ke^{-j\tau\omega}}{\pi A}=4\omega^2-j\omega(4-\omega^2)$$

代入
$$\omega=1,\quad A=10Ac=\frac{10}{4}$$
$$\frac{40Ke^{-j\tau}}{10\pi/4}=\frac{16Ke^{-j\tau}}{\pi}=4-j3=5e^{-j\arctan\frac{3}{4}}$$

比较模相角
$$\begin{cases}K=\frac{5}{16}\pi=0.3125\pi=0.9817\\\tau=\arctan\frac{3}{4}=0.6435\end{cases}$$

西北工业大学2005年硕士研究生入学考试 自动控制原理试题解答

一、(1) ① $sX_1=k_1[R-c-\beta X_3]$ $\quad\frac{X_1}{R-c-\beta X_3}=\frac{k_1}{s}$ ② $X_2=\tau sR$ $\quad\frac{X_2}{R}=\tau s$

③ $[Ts+1]X_3=X_1+X_2$ $\quad\frac{X_3}{X_1+X_2}=\frac{1}{Ts+1}$ ④ $sc=k_2X_3$ $\quad\frac{c}{X_3}=\frac{k_2}{s}$

结构图如图 D.2.1 所示。

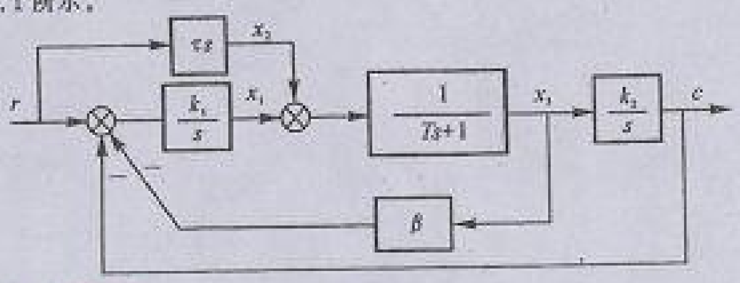

图 D.2.1

(2) 由梅逊公式

$$\Phi(s)=\frac{C(s)}{R(s)}=\frac{k_2\tau s^2+k_1k_2}{Ts^3+s^2+k_1\beta s+k_1k_2}$$

$$G(s)=\frac{K_0}{s(s+3)^2}\qquad\begin{cases}K=\frac{K_0}{9}\\V=1\end{cases}$$

① 渐近线
$$\begin{cases}\sigma_a=-2\\\varphi_a=\pm 60^\circ,180^\circ\end{cases}$$

② 分离点 $\frac{1}{d}+\frac{2}{d+3}=0\Rightarrow d=-1$

③ 与虚轴交点
$$D(s)=s^3+6s^2+9s+K_0=0$$
$$\begin{cases}\mathrm{Im}[D(j\omega)]=-\omega^3+9\omega=0\\\mathrm{Re}[D(j\omega)]=-6\omega^2+K_0=0\end{cases}\quad\begin{cases}\omega=3\\K_0=54,\quad K=\frac{54}{9}=6\end{cases}$$

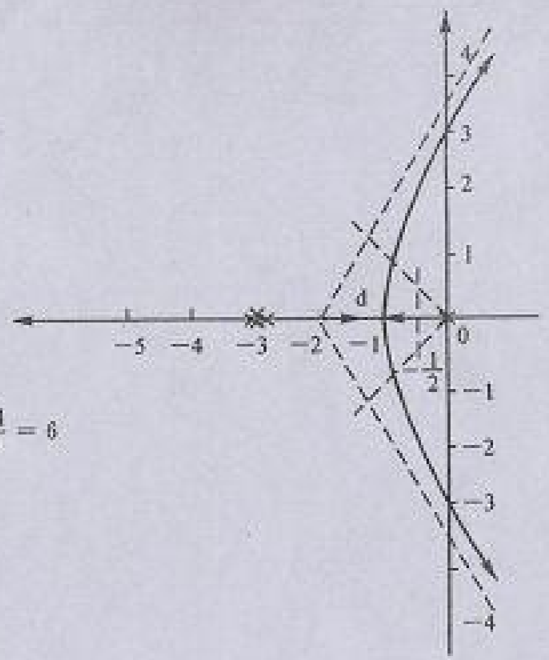

图 D.2.2

(2) 依题并由根轨迹，令 $\lambda_1=-0.5$

$K_{0\lambda_1}=|-0.5|\cdot|-0.5+0.3|^2=3.125$

$\xi=0.707$ 时，可设 $\lambda_{1,2}=-\sigma\pm j\sigma$

由根之和 $\qquad\lambda_3=-6+2\sigma$

$$D(s)=s^3+6s^2+9s+K_0=(s+\sigma+j\sigma)(s+\sigma-j\sigma)\times(s+6-2\sigma)=s^3+6s^2+(12\sigma-2\sigma^2)s+2\sigma(6-2\sigma)$$

比较系数
$$\begin{cases}\sigma^2-6\sigma+4.5=0\\K_0=2\sigma(6-2\sigma)\end{cases}$$

解出
$$\begin{cases}\sigma=0.87868\\K_0=6.5513\end{cases}$$

依题意有
$$3.125<K_0<6.5513$$
$$0.3472<K=\frac{K_0}{9}<0.728$$

(3) $e_{ss}\xrightarrow{r=t}\frac{1}{K}=\frac{9}{K_0}\xrightarrow{\text{临界稳定时 }K_0=54}\frac{9}{54}=\frac{1}{6}=0.1667$

三、依题意
$$\begin{cases}M_r=\frac{1}{2\xi\sqrt{1-\xi^2}}=1.1547\\\omega_r=\omega_n\sqrt{1-2\xi^2}=7.07\end{cases}$$

可解出
$$\begin{cases}\xi=0.5\\\omega_n=10\end{cases}\qquad\begin{cases}\sigma\%=16.3\%\\t_s=\frac{3.5}{\xi\omega_n}=\frac{3.5}{5}=0.7\end{cases}$$

(2)
$$G(s)=\frac{K}{s(Ts+1)}=\frac{\omega_n^2}{s(s+2\xi\omega_n)}=\frac{\omega_n/2\xi}{s\left(\frac{1}{2\xi\omega_n}s+1\right)}$$

比较得
$$\begin{cases}K=\frac{\omega_n}{2\xi}=10\\T=\frac{1}{2\xi\omega_n}=\frac{1}{10}\end{cases}$$

(3) 依题有 $G(s)=\frac{K}{s(Ts+1)}e^{-\tau s}=\frac{10}{s\left(\frac{s}{10}+1\right)}e^{-\tau s}$

概略绘制 $L(\omega)$ 曲线如图 D.2.3 所示，令相角裕度为 $0(\omega_c=10)$

$$\gamma=180^\circ-90^\circ-\arctan\frac{10}{10}-57.3^\circ\times10\tau=$$
$$180^\circ-90^\circ-45^\circ-573\tau=0$$

所以
$$\tau=\frac{45}{573}=0.0785$$

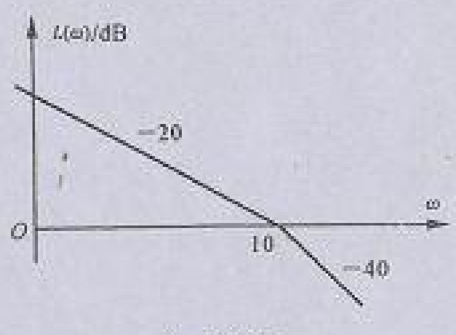

图 D.2.3

3.

$$G_0(s)=\frac{10}{(s+1)\left(\frac{s}{2}+1\right)\left(\frac{s}{10}+1\right)}$$

$$\omega_{c0}=\sqrt{2\times 10}=4.47$$

$$\gamma_0=180^\circ-\arctan\frac{4.47}{1}-\arctan\frac{4.47}{2}-\arctan\frac{4.47}{10}=180^\circ-77.4^\circ-65.9^\circ-24.1^\circ=12.6^\circ$$

(2) 依题校正后有

$$\Phi(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\begin{cases}\sigma\%=16.3\%\\ t_s=\dfrac{3.5}{\xi\omega_n}=0.7\end{cases}\quad 解出\quad \begin{cases}\xi=0.5\\ \omega_n=10\end{cases}$$

$$\Phi(s)=\frac{100}{s^2+10s+100}$$

$$G(s)=\frac{\Phi}{1-\Phi}=\frac{100}{s(s+10)}=\frac{10}{s\left(\frac{s}{10}+1\right)}$$

$$G_c(s)=\frac{G(s)}{G_0(s)}=\frac{10}{s\left(\frac{s}{10}+1\right)}\frac{(s+1)\left(\frac{s}{2}+1\right)\left(\frac{s}{10}+1\right)}{10}=\frac{(s+1)\left(\frac{s}{2}+1\right)}{5}=$$

$$\frac{s}{2}+1.5+\frac{1}{s}=K_D s+K_P=\frac{K_I}{s}$$

比较得 $K_D=\frac{1}{2}$， $K_P=1.5$， $K_I=1$

五、

$$G(z)=\frac{C^*(z)}{E^*(z)}=\frac{\mathscr{Z}\left[\frac{10(1-e^{-Ts})}{s^3}\right]}{1+\mathscr{Z}\left[\frac{5s(1-e^{-Ts})}{s^3}\right]}=\frac{10(1-z^{-1})\mathscr{Z}\left[\frac{1}{s^3}\right]}{1+5[1-z^{-1}]\mathscr{Z}\left[\frac{1}{s^2}\right]}=$$

$$\frac{10\left(\frac{z-1}{z}\right)\frac{T^2z(z+1)}{2(z-1)^3}}{1+5\left(\frac{z-1}{z}\right)\frac{Tz}{(z-1)^2}}\frac{5(z+1)T^2/(z-1)^2}{1+5T/(z-1)}=$$

$$\frac{5T^2(z+1)}{(z-1)(z-1+5T)}\xlongequal{T=0.2}\frac{0.2(z+1)}{z(z-1)}$$

$$\Phi_e(z)=\frac{E(z)}{R(z)}=\frac{1}{1+G(z)}=\frac{1}{1+\frac{0.2(z+1)}{z(z-1)}}=\frac{z(z-1)}{z^2-0.8z+0.2}$$

(1) 判断稳定性

$$D(z)=z^2-0.8z+0.2$$

$$\left.\begin{array}{l}D(1)=1-0.8+0.2=0.4>0\\ D(1)=1+0.8+0.2=2>0\\ |0.2|<|1|\end{array}\right\}闭环系统稳定$$

(2)

$$e^*(\infty)=\frac{TA}{K_v}$$

$$K_v=\lim_{z\to 1}(z-1)G(z)=\lim_{z\to 1}\frac{0.2(z+1)}{z}=0.4$$

所以

$$e^*(\infty)=\frac{0.2\times 1}{0.4}=0.5$$

$$\frac{-1}{N(A)}=\frac{-4}{3A^2}\quad\begin{cases}A=0\text{ 时},\dfrac{-1}{N(A)}=-\infty\\ A\to\infty\text{ 时},\dfrac{-1}{N(A)}\to 0\end{cases}$$

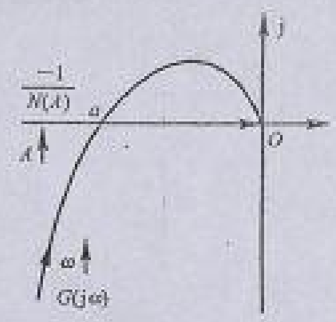

图 D.2.4

所以$\frac{-1}{N(A)}$曲线如图 D.2.4 所示。

绘出 $G(j\omega)$ 曲线，可见 a 不是自振点，求 a 点参数(A,ω)

令 $$N(A)G(j\omega)=\frac{3A^2}{4}\frac{8}{j\omega(1+j\omega)(2+j\omega)}=-1$$

$$6A^2=-j\omega(1+j\omega)(2+j\omega)=3\omega^2-j\omega(2-\omega^2)$$

比较有

$$\begin{cases}6A^2=3\omega^2\\ 2-\omega^2=0\end{cases}$$

解出

$$\begin{cases}A=\sqrt{\frac{1}{2}}\cdot\sqrt{2}=1\\ \omega=\sqrt{2}\end{cases}$$

依题及分析结果：

① 系统的自由运动形式有两种：稳定或发散。

② 当初始扰动幅值$\begin{cases}A>1\text{ 时，运动发散。}\\ A<1\text{ 时，运动收敛。}\end{cases}$

西北工业大学 2006 年硕士研究生入学考试

自动控制原理试题解答

一、依题有

$$my''+fy'+ky=x$$

$$\Phi(s)=\frac{y(s)}{x(s)}=\frac{1}{ms^2+fs+k}=\frac{1/m}{s^2+\frac{f}{m}s+\frac{k}{m}}=\frac{\frac{1}{k}\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$h(\infty)=0.5=\lim_{s\to 0}s\cdot\Phi(s)\cdot\frac{1}{s}=\frac{1}{k}\Rightarrow k=2$$

$$\begin{cases}\sigma\%=\dfrac{0.55-0.5}{0.5}=10\%=e^{-\xi\pi/\sqrt{1-\xi^2}}\\ t_p=\dfrac{\pi}{\sqrt{1-\xi^2}\omega_n}\end{cases}\Rightarrow\begin{cases}\xi=0.591\\ \omega_n=1.95\end{cases}$$

$$\begin{cases}k/m=\omega_n^2=1.95^2=3.8\\ f/m=2\xi\omega_n=2\times 0.591\times 1.95=2.305\end{cases}$$

解出 $k=2$， $m=0.526$， $f=1.213$

$$\Phi(s)=\frac{1.9}{s^2+2.3s+3.8}$$

二、(1) 依题可设 $$G(s)=\frac{abK}{s(s+a)(s+b)}\quad\begin{cases}K^*=abK\\ V=1\end{cases}$$

$$D(s)=s(s+a)(s+b)+abK=s^3+(a+b)s^2+abs+abK=0$$

$$\begin{cases}\mathrm{Re}[D(j\omega)]=-(a+b)\omega^2+Kab=0\\ \mathrm{Im}[D(j\omega)]=-\omega^3+ab\omega=0\end{cases}\Rightarrow\begin{cases}a=2\\ b=3\end{cases}$$

$$G(s)=\frac{6K}{\cdots}$$

4.

设 $\lambda_{1,2} = -\xi\omega_n \pm j\sqrt{1-\xi^2}\omega_n$

$$\sum_{i=1}^{3} = -5 = -2\xi\omega_n + \lambda_3 \Rightarrow \lambda_3 \xrightarrow{\xi=0.5} \omega_n - 5 =$$

$$(s^2+2\xi\omega_n s+\omega_n^2)(s+5-\omega_n)$$

$$D(s) \xrightarrow{\xi=0.5} s^3+5s^2+5\omega_n s+\omega_n^2(5-\omega_n)$$

比较 $\begin{cases}5\omega_n = 6\\ \omega_n^2(5-\omega_n) = 6K\end{cases}$

解出 $\omega_n = 1.2,\quad K = 0.912$

故 $\begin{cases}\lambda_{1,2} = -\xi\omega_n \pm j\sqrt{1-\xi^2}\omega_n = -0.6 \pm j1.039\ 2\\ \lambda_3 = -5+\omega_n = -3.8\end{cases}$ $\begin{cases}\sigma\% = 16.3\%\\ t_s = \dfrac{3.5}{0.6} = 5.83\end{cases}$

(3) $K = 0.912 = \omega_c$

$$\gamma = 180^\circ - 90^\circ - \arctan\frac{0.912}{2} - \arctan\frac{0.912}{3} = 90^\circ - 24.5^\circ - 16.9^\circ = 48.6^\circ$$

三、(1) $G(s) = \dfrac{-Ka(s-1)}{s(1+\frac{1}{3}s)} = \dfrac{-3Ka(s-1)}{s(s+3)}$ $\begin{cases}K_k = -aK\\ V = 1\end{cases}$

(Ⅰ) $a<0$ 时，画 180° 根轨迹（见图 D.3.1），此时系统不稳定。

(Ⅱ) $a>0$ 时，画 0° 根轨迹如图 D.3.2 所示。

① 分离点 $\dfrac{1}{d}+\dfrac{1}{d+3} = \dfrac{1}{d-1}$

整理有 $d^2-2d-3=0$

解出 $\begin{cases}d_1 = -1\\ d_2 = 3\end{cases}$ $\begin{cases}Ka_{d_1} = \dfrac{|d_1|\cdot|d_1+3|}{3|d_1-1|} = \dfrac{1}{3}\\ Ka_{d_2} = \dfrac{|d_2|\cdot|d_2+3|}{3|d_2+1|} = 3\end{cases}$ ①

② 与虚轴交点 $D(s) = s^2+(3-3Ka)s+3Ka = 0$

$$\begin{cases}\mathrm{Re}[D(j\omega)] = -\omega^2+3Ka = 0\\ \mathrm{Im}[D(j\omega)] = (3-3Ka)\omega = 0\end{cases} \Rightarrow \begin{cases}\omega = \sqrt{3}\\ Ka - 1\end{cases} \quad ②$$

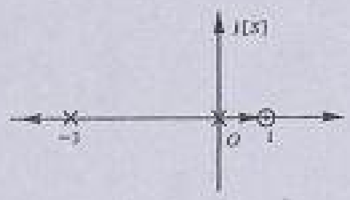

图 D.3.1

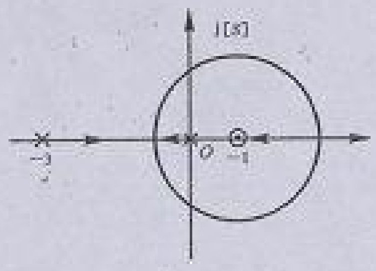

图 D.3.2

(2) $\Phi(s) = \dfrac{-3K(s-1)}{s(s+3)-3Ka(s-1)}$

$$h(\infty) = \lim_{s\to 0} s\cdot\frac{1}{s}\cdot\Phi(s) = \lim_{s\to 0}\frac{-3K(s-1)}{s(s+3)-3Ka(s-1)} = \frac{1}{a} = 2 \quad ③$$

$$a = \frac{1}{2}$$

由根轨迹确定出满足的条件 $\dfrac{1}{3} < Ka < 1$

即 $\dfrac{2}{3} < K < 2$

四、(1) 依题可设 $G_0(s) = \dfrac{\omega_{n0}^2}{s(s+2\xi_0\omega_{n0})}$ $\begin{cases}K_0 = \dfrac{\omega_{n0}}{2\xi_0}\\ V_0 = 1\end{cases}$

$$\Phi(s) = \frac{\omega_{n0}^2}{s^2+2\xi_0\omega_{n0}s+\omega_{n0}^2}$$

$\omega = 2$ 时，$\begin{cases}|\Phi_0(j2)| = \dfrac{1}{2\xi_0} = \dfrac{2}{1} = 2\\ |\Phi_0(j2)| = -90^\circ\end{cases}$ $\begin{cases}\omega_{n0} = 2\\ \xi_0 = \dfrac{1}{4} = 0.25\end{cases}$

$$G_0(s) = \frac{4}{s(s+1)}$$

(2) 可设 $G(s) = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$ $\begin{cases}\ \\ V = 1\end{cases}$

$$\Phi(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

依题意 $\begin{cases}e_{ss} = \dfrac{1}{K} = \dfrac{2\xi}{\omega_n} = 0.25\\ \sigma\% = e^{-\xi\pi/\sqrt{1-\xi^2}} = 16.3\%\end{cases}$ $\begin{cases}K = 4\\ \xi = 0.5\\ \omega_n = 4\end{cases}$

$$G(s) = \frac{4^2}{s(s+4)} = \frac{4}{s\left(\frac{s}{4}+1\right)}$$

依 $L(\omega)$ $\begin{cases}\omega_c = 4\\ \gamma = 180^\circ - 90^\circ - \arctan\dfrac{4}{4} = 45^\circ\end{cases}$

(3) $G_c(s) = \dfrac{G(s)}{G_0(s)} = \dfrac{4}{s\left(\frac{s}{4}+1\right)}\cdot\dfrac{s(s+1)}{4} = \dfrac{s+1}{\frac{s}{4}+1}$（超前校正）

五、 $G(z) = \mathscr{Z}\left[\dfrac{K}{s(s+1)}\right] = K\mathscr{Z}\left[\dfrac{1}{s}-\dfrac{1}{s+1}\right] = K\left[\dfrac{z}{z-1}-\dfrac{z}{z-e^{-T}}\right] = \dfrac{(1-e^{-T})Kz}{(z-1)(z-e^{-T})}$

(1) 由静态误差系数法

$$K_v = \lim_{z\to 1}(z-1)G(z) = \lim_{z\to 1}\frac{(1-e^{-T})Kz}{(z-e^{-T})} = K$$

$$e_{ss} = \frac{AT}{K_v} = \frac{T}{K} = 0.1T \quad \Rightarrow K = 10$$

(2) $K = 10$ 时，

$$\Phi(z) = \frac{G(z)}{1+G(z)} = \frac{(1-e^{-T})Kz}{(z-1)(z-e^{-T})+(1-e^{-T})Kz}$$

$$D(z) = (z-1)(z-e^{-T})+(1-e^{-T})Kz \xrightarrow{K=10} Z^2+[-(1+e^{-T})+10(1-e^{-T})]z+e^{-T} =$$
$$z^2+(9-11e^{-T})z+e^{-T}$$

由朱利判据 $\begin{cases}D(1) = K(1-e^{-T}) = 10(1-e^{-T}) > 0 & ①\\ D(-1) = 2(1+e^{-T})-10(1-e^{-T}) = 12e^{-T}-8 > 0 & ②\\ e^{-T} < 1 & ③\end{cases}$

①、③ $e^{-T} < 1 \Rightarrow T > 0$

② $e^{-T} > \dfrac{8}{12} \Rightarrow T < \ln\dfrac{12}{8} = 0.405\ 5$

故 $0 < T < 0.405\ 5$

六、如图 D.3.3 所示。

$G(s) = \dfrac{10}{s(s+1)(s+2)}$ $\begin{cases}K = 5\\ V = 1\end{cases}$

由 $N(A)G(j\omega) = -1$

$$\frac{4M}{\pi A}\cdot\frac{10}{j\omega(1+j\omega)(2+j\omega)} = -1$$

$$\frac{40M}{\pi A} = -j\omega[2-\omega^2+j3\omega] = 3\omega^2 - j\omega(2-\omega^2)$$

比较 $\begin{cases}\omega^2 = 2\\ \dfrac{40M}{\pi A} = 3\omega^2 = 6\end{cases}$

解出 $\begin{cases}\omega = \sqrt{2}\\ A = \dfrac{40M}{6\pi} = 2.123\ M\end{cases}$

依结构图有

$\dfrac{A}{A_c} = \dfrac{5}{\sqrt{\omega^2+2^2}}\Big|_{\omega=\sqrt{2}} = \dfrac{5}{\sqrt{6}} = 2.04$

故 $A_c = \dfrac{A}{2.04} = \dfrac{2.123M}{2.04} = 1.04M$