3.

$$G_0(s) = \frac{10}{(s+1)\left(\frac{s}{2}+1\right)\left(\frac{s}{10}+1\right)}$$

$$\omega_{c0} = \sqrt{2\times 10} = 4.47$$

$$\gamma_0 = 180^\circ - \arctan\frac{4.47}{1} - \arctan\frac{4.47}{2} - \arctan\frac{4.47}{10} = 180^\circ - 77.4^\circ - 65.9^\circ - 24.1^\circ = 12.6^\circ$$

(2) 依题校正后有

$$\Phi(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\begin{cases} \sigma\% = 16.3\% \\ t_s = \dfrac{3.5}{\xi\omega_n} = 0.7 \end{cases} \quad 解出 \quad \begin{cases} \xi = 0.5 \\ \omega_n = 10 \end{cases}$$

$$\Phi(s) = \frac{100}{s^2 + 10s + 100}$$

$$G(s) = \frac{\Phi}{1-\Phi} = \frac{100}{s(s+10)} = \frac{10}{s\left(\frac{s}{10}+1\right)}$$

$$G_c(s) = \frac{G(s)}{G_0(s)} = \frac{10}{s\left(\frac{s}{10}+1\right)} \frac{(s+1)\left(\frac{s}{2}+1\right)\left(\frac{s}{10}+1\right)}{10} = \frac{(s+1)\left(\frac{s}{2}+1\right)}{5} =$$

$$\frac{s}{2} + 1.5 + \frac{1}{s} = K_D s + K_P = \frac{K_I}{s}$$

比较得 $K_D = \frac{1}{2}$，$K_P = 1.5$，$K_I = 1$

五、

$$G(z) = \frac{C^*(z)}{E^*(z)} = \frac{\mathcal{Z}\left[\frac{10(1-e^{-Ts})}{s^3}\right]}{1+\mathcal{Z}\left[\frac{5s(1-e^{-Ts})}{s^3}\right]} = \frac{10(1-z^{-1})\mathcal{Z}\left[\frac{1}{s^3}\right]}{1+5[1-z^{-1}]\mathcal{Z}\left[\frac{1}{s^2}\right]} =$$

$$\frac{10\left(\frac{z-1}{z}\right)\frac{T^2 z(z+1)}{2(z-1)^3}}{1+5\left(\frac{z-1}{z}\right)\frac{Tz}{(z-1)^2}} \frac{5(z+1)T^2/(z-1)^2}{1+5T/(z-1)} =$$

$$\frac{5T^2(z+1)}{(z-1)(z-1+5T)} \xrightarrow{T=0.2} \frac{0.2(z+1)}{z(z-1)}$$

$$\Phi_e(z) = \frac{E(z)}{R(z)} = \frac{1}{1+G(z)} = \frac{1}{1+\frac{0.2(z+1)}{z(z-1)}} = \frac{z(z-1)}{z^2-0.8z+0.2}$$

(1) 判断稳定性

$$D(z) = z^2 - 0.8z + 0.2$$

$$\left.\begin{array}{l} D(1) = 1-0.8+0.2 = 0.4 > 0 \\ D(1) = 1+0.8+0.2 = 2 > 0 \\ |0.2| < |1| \end{array}\right\} 闭环系统稳定$$

(2)

$$e^*(\infty) = \frac{TA}{K_v}$$

$$K_v = \lim_{z\to 1}(z-1)G(z) = \lim_{z\to 1}\frac{0.2(z+1)}{z} = 0.4$$

所以

$$e^*(\infty) = \frac{0.2\times 1}{0.4} = 0.5$$

$$\frac{-1}{N(A)} = \frac{-4}{3A^2} \quad \begin{cases} A = 0 时，\dfrac{-1}{N(A)} = -\infty \\ A \to \infty 时，\dfrac{-1}{N(A)} \to 0 \end{cases}$$

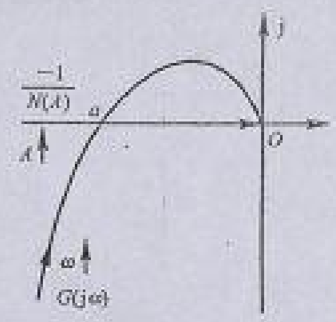


图 D.2.4

所以 $\frac{-1}{N(A)}$ 曲线如图 D.2.4 所示。

绘出 $G(j\omega)$ 曲线，可见 a 不是自振点，求 a 点参数 (A,ω)

令 $$N(A)G(j\omega) = \frac{3A^2}{4}\frac{8}{j\omega(1+j\omega)(2+j\omega)} = -1$$

$$6A^2 = -j\omega(1+j\omega)(2+j\omega) = 3\omega^2 - j\omega(2-\omega^2)$$

比较有

$$\begin{cases} 6A^2 = 3\omega^2 \\ 2-\omega^2 = 0 \end{cases}$$

解出

$$\begin{cases} A = \sqrt{\frac{1}{2}}\cdot\sqrt{2} = 1 \\ \omega = \sqrt{2} \end{cases}$$

依题及分析结果：

① 系统的自由运动形式有两种：稳定或发散。

② 当初始扰动幅值 $\begin{cases} A > 1 时，运动发散。\\ A < 1 时，运动收敛。\end{cases}$

西北工业大学 2006 年硕士研究生入学考试

自动控制原理试题解答

一、依题有 $$my'' + fy' + ky = x$$

$$\Phi(s) = \frac{y(s)}{x(s)} = \frac{1}{ms^2+fs+k} = \frac{1/m}{s^2+\frac{f}{m}s+\frac{k}{m}} = \frac{\frac{1}{k}\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$h(\infty) = 0.5 = \lim_{s\to 0} s\cdot\Phi(s)\cdot\frac{1}{s} = \frac{1}{k} \Rightarrow k = 2$$

$$\begin{cases} \sigma\% = \dfrac{0.55-0.5}{0.5} = 10\% = e^{-\xi\pi/\sqrt{1-\xi^2}} \\ t_p = \dfrac{\pi}{\sqrt{1-\xi^2}\,\omega_n} \end{cases} \Rightarrow \begin{cases} \xi = 0.591 \\ \omega_n = 1.95 \end{cases}$$

$$\begin{cases} k/m = \omega_n^2 = 1.95^2 = 3.8 \\ f/m = 2\xi\omega_n = 2\times 0.591\times 1.95 = 2.305 \end{cases}$$

解出 $k = 2$，$m = 0.526$，$f = 1.213$

$$\Phi(s) = \frac{1.9}{s^2+2.3s+3.8}$$

二、(1) 依题可设 $$G(s) = \frac{abK}{s(s+a)(s+b)} \quad \begin{cases} K^* = abK \\ V = 1 \end{cases}$$

$$D(s) = s(s+a)(s+b) + abK = s^3 + (a+b)s^2 + abs + abK = 0$$

$$\begin{cases} \mathrm{Re}[D(j\omega)] = -(a+b)\omega^2 + Kab = 0 \\ \mathrm{Im}[D(j\omega)] = -\omega^3 + ab\omega = 0 \end{cases} \Rightarrow \begin{cases} a = 2 \\ b = 3 \end{cases}$$

$$G(s) = \frac{6K}{\text{[illegible]}}$$

4.

设 $\lambda_{1,2}=-\xi\omega_n\pm j\sqrt{1-\xi^2}\omega_n$

$$\sum_{i=1}^{3}=-5=-2\xi\omega_n+\lambda_3\Rightarrow\lambda_3\xrightarrow{\xi=0.5}\omega_n-5=$$

$$(s^2+2\xi\omega_n s+\omega_n^2)(s+5-\omega_n)$$

$$D(s)\xrightarrow{\xi=0.5}s^3+5s^2+5\omega_n s+\omega_n^2(5-\omega_n)$$

比较 $\begin{cases}5\omega_n=6\\ \omega_n^2(5-\omega_n)=6K\end{cases}$

解出 $\omega_n=1.2,\quad K=0.912$

故 $\begin{cases}\lambda_{1,2}=-\xi\omega_n\pm j\sqrt{1-\xi^2}\omega_n=-0.6\pm j1.039\ 2\\ \lambda_3=-5+\omega_n=-3.8\end{cases}$ $\begin{cases}\sigma\%=16.3\%\\ t_s=\dfrac{3.5}{0.6}=5.83\end{cases}$

(3) $K=0.912=\omega_c$

$$\gamma=180^\circ-90^\circ-\arctan\frac{0.912}{2}-\arctan\frac{0.912}{3}=90^\circ-24.5^\circ-16.9^\circ=48.6^\circ$$

三、(1) $G(s)=\dfrac{-Ka(s-1)}{s(1+\frac{1}{3}s)}=\dfrac{-3Ka(s-1)}{s(s+3)}$ $\begin{cases}K_k=-aK\\ V=1\end{cases}$

(Ⅰ) $a<0$ 时，画 180° 根轨迹（见图 D.3.1），此时系统不稳定。

(Ⅱ) $a>0$ 时，画 0° 根轨迹如图 D.3.2 所示。

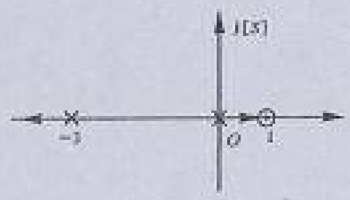


图 D.3.1

① 分离点 $\dfrac{1}{d}+\dfrac{1}{d+3}=\dfrac{1}{d-1}$

整理有 $d^2-2d-3=0$

解出 $\begin{cases}d_1=-1\\ d_2=3\end{cases}$ $\begin{cases}Ka_{d_1}=\dfrac{|d_1|\cdot|d_1+3|}{3|d_1-1|}=\dfrac{1}{3}\\ Ka_{d_2}=\dfrac{|d_2|\cdot|d_2+3|}{3|d_2+1|}=3\end{cases}$ ①

② 与虚轴交点 $D(s)=s^2+(3-3Ka)s+3Ka=0$

$$\begin{cases}\mathrm{Re}[D(j\omega)]=-\omega^2+3Ka=0\\ \mathrm{Im}[D(j\omega)]=(3-3Ka)\omega=0\end{cases}\Rightarrow\begin{cases}\omega=\sqrt{3}\\ Ka=1\end{cases}$$ ②

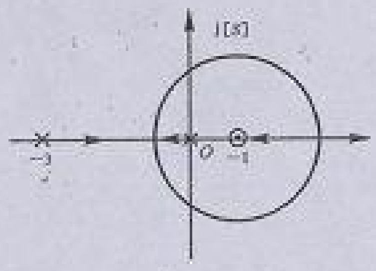


图 D.3.2

(2) $\Phi(s)=\dfrac{-3K(s-1)}{s(s+3)-3Ka(s-1)}$

$$h(\infty)=\lim_{s\to0}s\cdot\frac{1}{s}\cdot\Phi(s)=\lim_{s\to0}\frac{-3K(s-1)}{s(s+3)-3Ka(s-1)}=\frac{1}{a}=2$$ ③

$$a=\frac{1}{2}$$

由根轨迹确定出满足的条件 $\dfrac{1}{3}<Ka<1$

即 $\dfrac{2}{3}<K<2$

四、(1) 依题可设 $G_0(s)=\dfrac{\omega_{n0}^2}{s(s+2\xi_0\omega_{n0})}$ $\begin{cases}K_0=\dfrac{\omega_{n0}}{2\xi_0}\\ V_0=1\end{cases}$

$$\Phi(s)=\frac{\omega_{n0}^2}{s^2+2\xi_0\omega_{n0}s+\omega_{n0}^2}$$

$\omega=2$ 时，$\begin{cases}|\Phi_0(j2)|=\dfrac{1}{2\xi_0}=\dfrac{2}{1}=2\\ |\Phi_0(j2)|=-90^\circ\end{cases}$ $\begin{cases}\omega_{n0}=2\\ \xi_0=\dfrac{1}{4}=0.25\end{cases}$

$$G_0(s)=\frac{4}{s(s+1)}$$

(2) 可设 $G(s)=\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$ $\begin{cases}\ldots\\ V=1\end{cases}$

$$\Phi(s)=\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

依题意 $\begin{cases}e_{ss}=\dfrac{1}{K}=\dfrac{2\xi}{\omega_n}=0.25\\ \sigma\%=e^{-\xi\pi/\sqrt{1-\xi^2}}=16.3\%\end{cases}$ $\begin{cases}K=4\\ \xi=0.5\\ \omega_n=4\end{cases}$

$$G(s)=\frac{4^2}{s(s+4)}=\frac{4}{s\left(\frac{s}{4}+1\right)}$$

依 $L(\omega)$ $\begin{cases}\omega_c=4\\ \gamma=180^\circ-90^\circ-\arctan\dfrac{4}{4}=45^\circ\end{cases}$

(3) $G_c(s)=\dfrac{G(s)}{G_0(s)}=\dfrac{4}{s\left(\frac{s}{4}+1\right)}\cdot\dfrac{s(s+1)}{4}=\dfrac{s+1}{\frac{s}{4}+1}$（超前校正）

五、 $G(z)=\mathscr{Z}\left[\dfrac{K}{s(s+1)}\right]=K\mathscr{Z}\left[\dfrac{1}{s}-\dfrac{1}{s+1}\right]=K\left[\dfrac{z}{z-1}-\dfrac{z}{z-e^{-T}}\right]=\dfrac{(1-e^{-T})Kz}{(z-1)(z-e^{-T})}$

(1) 由静态误差系数法

$$K_v=\lim_{z\to1}(z-1)G(z)=\lim_{z\to1}\frac{(1-e^{-T})Kz}{(z-e^{-T})}=K$$

$$e_{ss}=\frac{AT}{K_v}=\frac{T}{K}=0.1T\quad\Rightarrow K=10$$

(2) $K=10$ 时，

$$\Phi(z)=\frac{G(z)}{1+G(z)}=\frac{(1-e^{-T})Kz}{(z-1)(z-e^{-T})+(1-e^{-T})Kz}$$

$$D(z)=(z-1)(z-e^{-T})+(1-e^{-T})Kz\xrightarrow{K=10}Z^2+[-(1+e^{-T})+10(1-e^{-T})]z+e^{-T}=$$
$$z^2+(9-11e^{-T})z+e^{-T}$$

由朱利判据 $\begin{cases}D(1)=K(1-e^{-T})=10(1-e^{-T})>0 & ①\\ D(-1)=2(1+e^{-T})-10(1-e^{-T})=12e^{-T}-8>0 & ②\\ e^{-T}<1 & ③\end{cases}$

①、③ $e^{-T}<1\Rightarrow T>0$

② $e^{-T}>\dfrac{8}{12}\Rightarrow T<\ln\dfrac{12}{8}=0.405\ 5$

故 $0<T<0.405\ 5$

六、如图 D.3.3 所示。

$G(s)=\dfrac{10}{s(s+1)(s+2)}$ $\begin{cases}K=5\\ V=1\end{cases}$

$\dfrac{A}{A_c}=\dfrac{5}{\sqrt{\omega^2+2^2}}\Big|_{\omega=\sqrt{2}}=\dfrac{5}{\sqrt{6}}=2.04$

由 $N(A)G(j\omega)=-1$

$$\frac{4M}{\pi A}\cdot\frac{10}{j\omega(1+j\omega)(2+j\omega)}=-1$$

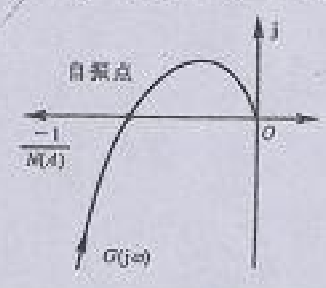
$$\frac{40M}{\pi A}=-j\omega[2-\omega^2+j3\omega]=3\omega^2-j\omega(2-\omega^2)$$

故 $A_c=\dfrac{A}{2.04}=\dfrac{2.123M}{2.04}=1.04M$

系统自由响应一定自振

比较 $\begin{cases}\omega^2=2\\ \dfrac{40M}{\pi A}=3\omega^2=6\end{cases}$

解出 $\begin{cases}\omega=\sqrt{2}\\ A=\dfrac{40M}{6\pi}=2.123\ M\end{cases}$



依结构图有